1) Show that $\neg(p \lor \neg q)$ and $q \land \neg p$ are equivalent.

2) Let p, q, r be the following propositions:
p : You get an A in this class
q : You understand every exercise in the book
r : You like logic puzzles

Translate the statements below into propositional logic.

a) You like logic puzzles if you do every exercise in the book.
b) For you to get an A in the class it is sufficient that you understand every exercise in the book.
c) For you to get an A in this class it is necessary for you to like logic puzzles.
d) You get an A in this class even though you dislike logic puzzles or you do not understand every exercise in the book.

3) Use De Morgan's laws to show $\neg(p \land q) \land p \to \neg q$.

4) Prove that if n^2 is an even integer, n is an even integer. What type of proof technique did you use?

5) State the rules of *modus ponens* and *modus tollens* for an implication.

6) Prove via contradiction that if you consider 8 days at least 2 of them must fall on the same day of the week.

7) Prove there is exists a number n such that n is the product of two prime numbers. Did you use a constructive or nonconstructive existence proof?

8) State the two DeMorgan's laws that hold for the relations between sets A and B. Then generalize each law to consider three sets, A, B, and C.

9) Are the following implications/arguments valid? If they are invalid, what is the reason?
a) If 1+1=3 then today is Thursday. Is this a *vacuous* or *trivial* proof?
b) If today is Thursday, then I will go to the beach. I am at the beach so today is Thursday.

c) There exists a student who likes discrete math. John is a student therefore John likes discrete math.
d) There does not exist a student who likes doing homework, therefore all students dislike doing homework.

10) Prove that if x and y are both rational, than x/y is rational.

11) Are the following bijections? If not, explain why.
a) $f : \mathbb{Z} \to \Re, f(x) = 1/x$.
a) $f : \mathbb{Z}^+ \to \Re, f(x) = 1/x$.

12) Show explicitly how to derive a simple formula for $\sum_{i=2}^{n} i$ where n is an odd number.

13) Prove that if n is an integer, $n = \lfloor n/2 \rfloor + \lceil n/2 \rceil$.

14) Prove that if x is rational and y is irrational, then xy is irrational.

15) What is the power set of the set $A = \{1, 2, 3\}$. What is the cardinality of the power set of a general set S?