Lecture 3, April 9, 2019

Announcement:

- Hw1 due Weds April 17
  → Gradescope
  (include a cover page on canvas)

Recap:

- Proposition: declaritive statement which is
  True(T) or False(F), but not both or either.
  typically denoted lowercase $p, q, r, s, \ldots$

- Proposition operators

  $\neg$ not (unary), e.g. $\neg p$

  $\wedge$ and, $p \wedge q$
  $\vee$ or, $p \vee q$
  $\oplus$ exclusive or, $p \oplus q$
  } binary

- Truth tables to assess T/F values of compound propositions.

- A <u>tautology</u> is always true, e.g. $p \vee \neg p$, $p \oplus \neg p$

- A <u>contradiction</u> is logically inconsisent, e.g. $p \wedge \neg p$

e.g. q = "This statement is false" is a contradiction.

statement asserts $q = F$, but then $q = T$; $q \wedge \neg q$

If we assign $q = T$, then $q = F$; $q \wedge \neg q$

- The implication $p \to q$ The cornerstone of logic

"if p then q"

If $p = T$ then $q = T$

If $p = F$ then no info about q (q can be T or F)

If $q = T$, then no infor about p (p can be T or F)

If $\neg q = T$ ($q = F$) then $p = F$

contrapositive: $\neg q \to \neg p$

Language

- p is sufficient for q
- q is necessary for p

- Common fallacies "affirming the consequent"
  (Asserts that $q = T$ implies that $p = T$) ← Fallacy!

  p = "You use brand X toothpaste"

  q = "You are attractive w/ white teeth"

- The biimplication $p \leftrightarrow q$

  if $p = T$ then $q = T$
  If $p = F$ the $q = F$
  If $q = T$ then $p = T$
  If $q = F$ then $p = F$

  - p is necessary & sufficient for q
  - q " " " " " p
  - p if and only if q
  - q if and only if p

  $\Big\}$ "iff"

- Logical equivalences (see Table 6, Sec 1.3).

  Most important are De Morgan's Laws

  1) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

  2) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

  Notation $\equiv$, $\Longleftrightarrow$ mean logically equiv.

- Translating from/between English language & prop. statements

  Hint: make the symbol for the proposition associated w/ the statement

  e.g. m = "I like math"

  j = "I am a junior"

  - Understand synonyms:

    Unless, except if, either, sufficient, necessary, but, etc.

  - If there is any "if-statement", that if-statement proposition is the "p" part of $p \to q$.

    (see bottom of handout 1 for an example).

- Predicate logic allows for more flexible, general statements.

  $P(x)$ or $P(x, y, z)$ or $P(x_1, x_2, \ldots x_n)$

  predicate

  subject

  P is a function that takes an n-tuple as it's argument and generates a proposition.

  e.g. Allows us to translate an algebra statement to a proposition given the value of x.

TODAY: Universal quantifiers for predicate calculus.

→ quantify the range/values of x for which $P(x) = T$.

- Domain of discourse, U. (specifies what possible values x can take on
  Universe of discourse

  e.g. U = integers
  U = real numbers
  U = students in class

- $\forall x$ "for all x in U"

- $\exists x$ "there exists an x in U"

e.g.

$\forall x\, P(x)$ means $P(x) = T$ for all x in U.

$\exists x\, P(x)$ means there is at least one x in U for which $P(x) = T$.

variants

$\neg \forall x$, $\neg \exists x$, ... etc.

... Onto lecture 3 slides ...