Lec 18.

Announcements

- Please fill out evals.

Final Exam info

- Friday June 7, 1-3 pm, 123 Sciences Lec Hall
- Two sheets (front & back, 8.5 x 11" paper)
- Focus is on

$$\boxed{\begin{array}{ll} \text{- Handouts} & 7-13 \\ \text{- Hws} & 4-6. \end{array}}$$

- Review sessions:
  - Weds, 6:10-9pm, Keibler Hall 3
  - In-class Thurs
  - CS tutoring Friday 10am-noon, location TBD.

* Leverage the stress response *

Counting recap: Chap 6.1 - 6.3

Combinations (sets); $\boxed{C(n,r) = n! / r!(n-r!)}$

# of distinct sets of size r from n elements.

(recall "n,r" are "dummy variables"

Permutation (sequence); $\boxed{P(n,r) = n! / (n-r)!}$

# of distinct sequences of length r from n elements.

(sequence = list where each of ~~the~~ r items appears once).

The connection:

$$P(n,r) = \underbrace{C(n,r)}_{\text{\# of subsets of size } r} \; \underbrace{r!}_{\text{\# of distinct orderings for } r \text{ items}}$$

Recall $P(n,n) = n!$

$C(n,n) = 1$

Two classes of problems: for probabilities:

(I) order matters; winning a race, Lotto $P(n,m)$

(II) order doesn't matter; poker hands, Bingo, committee formation $C(n,m)$

Discrete probability; Likelihood of an outcome/event

Chap 7.1 - 7.2.

At simplest form assume:

- Finite number of outcomes

Sample space $S = \{s_1, s_2, s_3, \ldots s_m\}$

$|S| = \#$ of possible outcomes $= m$

- Uniform distribution; all s_i's are equally likely

$$p(s_i) = \frac{1}{|S|}$$

- More generally:

(i) $0 \leq p(s_i) \leq 1$

(ii) $\sum_{s_i \in S} p(s_i) = 1$

An "event" can involve multiple S_i's

(an event involves subset of s_i's that are compliant w/ that event).

$$\boxed{\text{Prob}(E) = \sum_{s_i \in E} p(s_i)}$$

Prob the event does not happen

$p(\bar{E}) \rightarrow \boxed{P(\bar{E}) = 1 - p(E)}$

Example system : a fair die

$$S = \{1, 2, 3, 4, 5, 6\}$$

Event 1: prob (3) = $1/6$

Event 2: prob of an even roll

$$p(\text{Evenroll}) = \sum_{s_i \in \text{Even}} p(s_i) = p(2) + p(4) + p(6)$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}.$$

Example system: a set of fair die.

$$S = \{ \{1,1\}, \{1,2\}, \{1,3\}, \ldots, \{6,6\} \}$$

$$|S| = 6^2 = 36$$

Event 1: The sum of the two die is 7.

$$E = \{ \{1,6\}, \{2,5\}, \{3,4\}, \{4,3\}, \{5,2\}, \{6,1\} \}$$

$$P(7) = \frac{6}{36}$$

More generally $\boxed{p(E) = \frac{|E|}{|S|}}$

Example system: "Lotto"

The probability of correct 6 number <u>sequence</u> from 40 numbers in total.

E = choose all 6 number correctly

$$P(E) = \frac{|E|}{|S|} = \frac{1}{P(40,6)} = \frac{1(34!)}{40!}$$

$$= \frac{1}{2.76 \text{ billion}}$$

A variant: unordered collection of the 6 numbers wins.

$$|S| = C(n,r) = C(40,6)$$

$$P(E) = \frac{1}{C(40,6)} = \frac{6!\,34!}{40!} = \frac{1}{3.8 \text{ million}}.$$

Prob of not winning = 1 − prob of winning.

$$P(\bar{E}) = 1 - P(E)$$

$$|S^{\text{ordred}}| = P(40,6) = 40!/34!$$

$$|S^{\text{unorderd}}| = C(40,6) = 40!/6!\,34!$$

Example system: 5-card Poker hands (order does not matter)

\# of card types 13; $\{2, 3, 4, \ldots 10, J, Q, K, A\}$

\# of suits 4; {clubs, spades, hearts, diamonds}

Prod rule: $13 \cdot 4 = 52$ cards in total.

\# of poker hands, $|S| = C(52, 5) = \dfrac{52!}{5!(47!)}$

Prob of any particular hand

$$p(E) = \frac{1}{C(52,5)}$$

More general classes of hands

Events e.g. "2-of-a-kind", "3-of-a-kind", "full house"

same type

5 cards

_____ _____ _____ _____ _____

Game plan to analyze the # of hands of this ~~ty~~ class.

i) Identify # of distinct types

ii) # of ways to assign types

iii) # of ways to assign suits

Example system: Drawing colored balls from a box

6 red balls
4 black

10 balls in total

# of balls drawn to be guaranteed at least one black one?

(generalized "pigeonhole")

- E be the event that if 4 balls are selected two are red & two are black?

# of outcomes $|S| = C(10,4) = \frac{10!}{4!\,6!} = 210$

Enumerate all outcomes:

$\underbrace{C(6,4)}_{\text{all red balls}}$; $\underbrace{C(6,3)}_{\text{3 reds}}\underbrace{C(4,1)}_{\text{1 black}}$; $\underbrace{C(6,2)}_{\text{2 red}}\underbrace{C(4,2)}_{\text{2 black}}$; $\underbrace{C(6,1)}_{\text{1 red}}\underbrace{C(4,3)}_{\text{3 black}}$; $\underbrace{C(4,4)}_{\text{4 black}}$

$15 + 20 \cdot 4 + 15 \cdot 6 + 6 \cdot 4 + 1$

$= 210.$

- Prob 2 red & 2 black selected $\frac{C(6,2)\,C(4,2)}{C(10,4)}$

- Prob that no more than two are red.

$1 - pr(3\text{reds}) - pr(4\text{reds})$