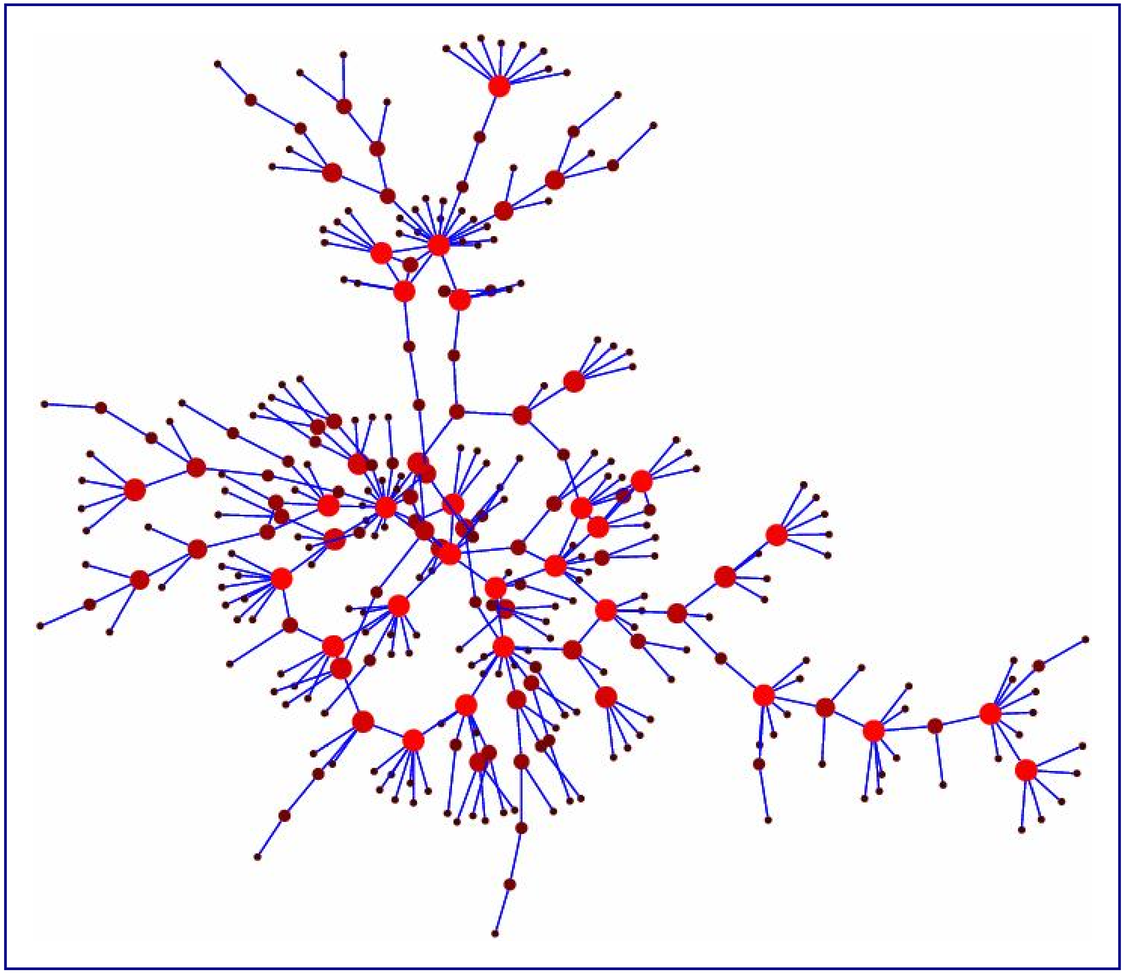
MAE 298, Lecture 12

Feb 25, 2008



"Flow on real-world networks II"

Topics

- Optimal allocation of facilities and transport networks:
  - Michael Gastner (SFI) and Mark Newman (U Mich)
- Network flows on road networks
  - I. User vs System Optimal
  - II. Braess' Paradox
  - Michael Zhang (UC Davis)
- Scale invariance in road networks:
  - Kalapala, Sanwalani, Clauset, Moore (UNM/SFI)
- Layered interacting networks:
  - Kurant and Thiran (EPFL)

Last time

Optimal design of spatial distribution systems:

(Download: Gastner.pdf)

Today

Flow on transportation networks:

(Download: Zhang.ppt)

Nash equilibrium versus optimal (Pareto optimum)
Prisoner's Dilemma

	Cooperate	Defect
Cooperate	3, 3	0, 5
Defect	5, 0	1, 1

- Blue Cooperates/Red Cooperates — Blue gets payout "3"
- Blue Cooperates/Red Defects – Blue gets "0"
- Blue Defects/Red Defects – Blue gets "1"
- Blue Defects/Red Cooperates – Blue gets "5"

– Average expected payout for defect is "3", for cooperate is "1.5". **Blue always chooses to Defect!** Likewise Red always chooses Defect.

– Both defect and get "1" (Nash), even though each would get a higher payout of "3" if they cooperated (Pareto).

Braess Paradox

- Dietrich Braess, 1968

  (Currently Prof of Math at Ruhr University Bochum, Germany)

- In a user-optimized network, when a new link is added, the change in equilibrium flows might result in a higher cost, implying that users were better off without that link.

- Recall Zhang notation, q_{ij} is overall traffic demand from node i to j. $t_a(\nu_a)$ is travel cost along link a, which is a function of total flow that link ν_a.

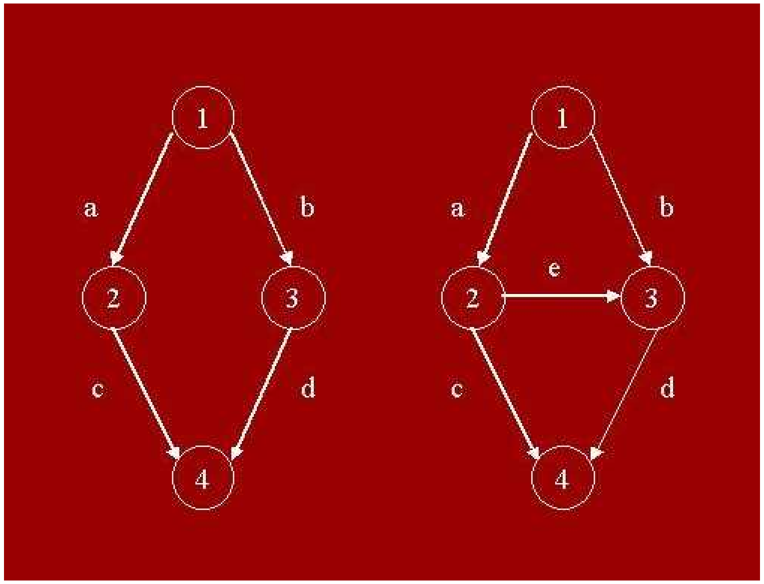
Getting from 1 to 4

Assume traffic demand $q_{14} = 6$. Originally 2 paths (a-c) and (b-d).

- $t_a(\nu_a) = 10\nu_a$
- $t_b(\nu_b) = \nu_b + 50$
- $t_c(\nu_c) = \nu_c + 50$
- $t_d(\nu_d) = 10\nu_d$

$\Longrightarrow$ Eqm: $\nu = 3$ on each link

$\mathbf{C_1 = C_2 = 83}$



Add new link with $t_e(\nu_e) = \nu_e + 10$

Now three paths:

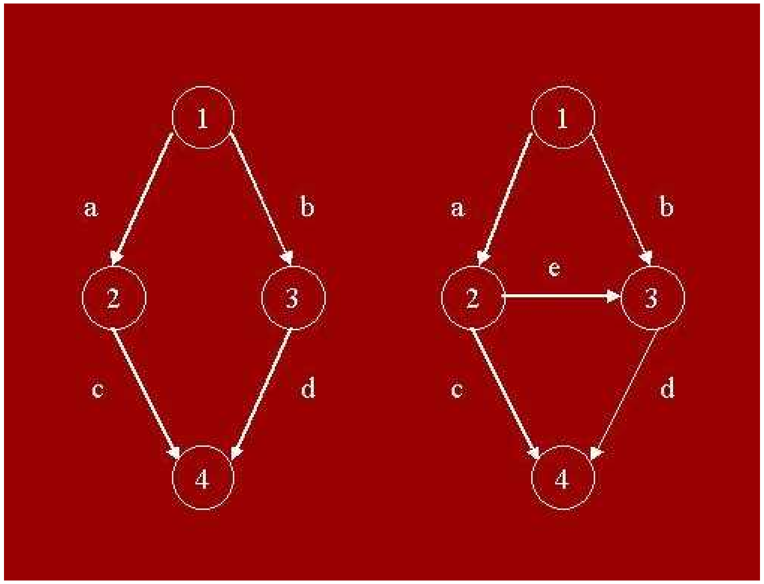
Path 3 (a - e - d), with $\nu_e = 0$ initially, so $C_3 = 0 + 10 + 0 = 10$

$C_3 < C_2$ and C_1 so new equilibrium needed.

- By inspection, shift one unit of flow form path 1 and from 2 respectively to path 3.

- Now all paths have flow $f_1 = f_2 = f_3 = 2$.

- Link flow $\nu_a = 4$, $\nu_b = 2$, $\nu_c = 2$, $\nu_d = 4$, $\nu_e = 2$.



$t_a = 40$, $t_b = 52$, $t_c = 52$, $t_d = 40$, $t_e = 12$.

$C_1 = t_a + t_c = \mathbf{92}$; $C_2 = t_b + t_d = \mathbf{92}$; $C_3 = t_a + t_e + t_d = \mathbf{92}$.

- $92 > 83$ **so just increased the travel cost!**

Braess paradox – Real-world examples

(from http://supernet.som.umass.edu/facts/braess.html)

- 42nd street closed in New York City. Instead of the predicted traffic gridlock, traffic flow actually improved.

- A new road was constructed in Stuttgart, Germany, traffic flow worsened and only improved after the road was torn up.

Braess Paradox – Applications to Internet traffic

Greg Valiant, Tim Roughgarden, Eva Tardos

e.g., "Braess's paradox in large random graphs", Proceedings of the 7th ACM conference on Electronic commerce, 2006.

- Removing edges from a network with "selfish routing" can decrease the latency incurred by traffic in an equilibrium flow.
- With high probability, (as the number of vertices goes to infinity), there is a traffic rate and a set of edges whose removal improves the latency of traffic in an equilibrium flow by a constant factor.
- Braess paradox found in random networks often (not just "classic" 4-node construction).

Braess paradox depends on parameter choices

- "Classic" 4-node Braess construction relies on details of q_{14} and the link travel cost functions, t_i.

- The example works because for small overall demand (q_{14}), links a and d are cheap. The new link e allows a path connecting them.

- If instead demand large, e.g. $q_{14} = 60$, now links a and d are costly! ($t_a = t_d = 600$ while $t_b = t_c = 110$). The new path a-e-d will always be more expensive so $\nu_e = 0$. No traffic will flow on that link. So Braess paradox does not arise for this choice of parameters.

How to avoid Braess?

- Back to Zhang presentation .... typically solve for optimal flows numerically using computers. Can test for a range of choices of traffic demand and link costs.

Our modern infrastructure

Layered, interacting networks

ELECTRICITY

Banking and Finance

Government

Emergency Response

Transportation

Oil and natural gas

Water

TELECOM

“Layered complex networks”

[M. Kurant and P. Thiran, “Layered Complex Networks”, Phys Rev Lett. 89, 2006.]

- Offer a simple formalism to think about two coexisting network topologies.
- The physical topology.
- And the virtual (application) topology.

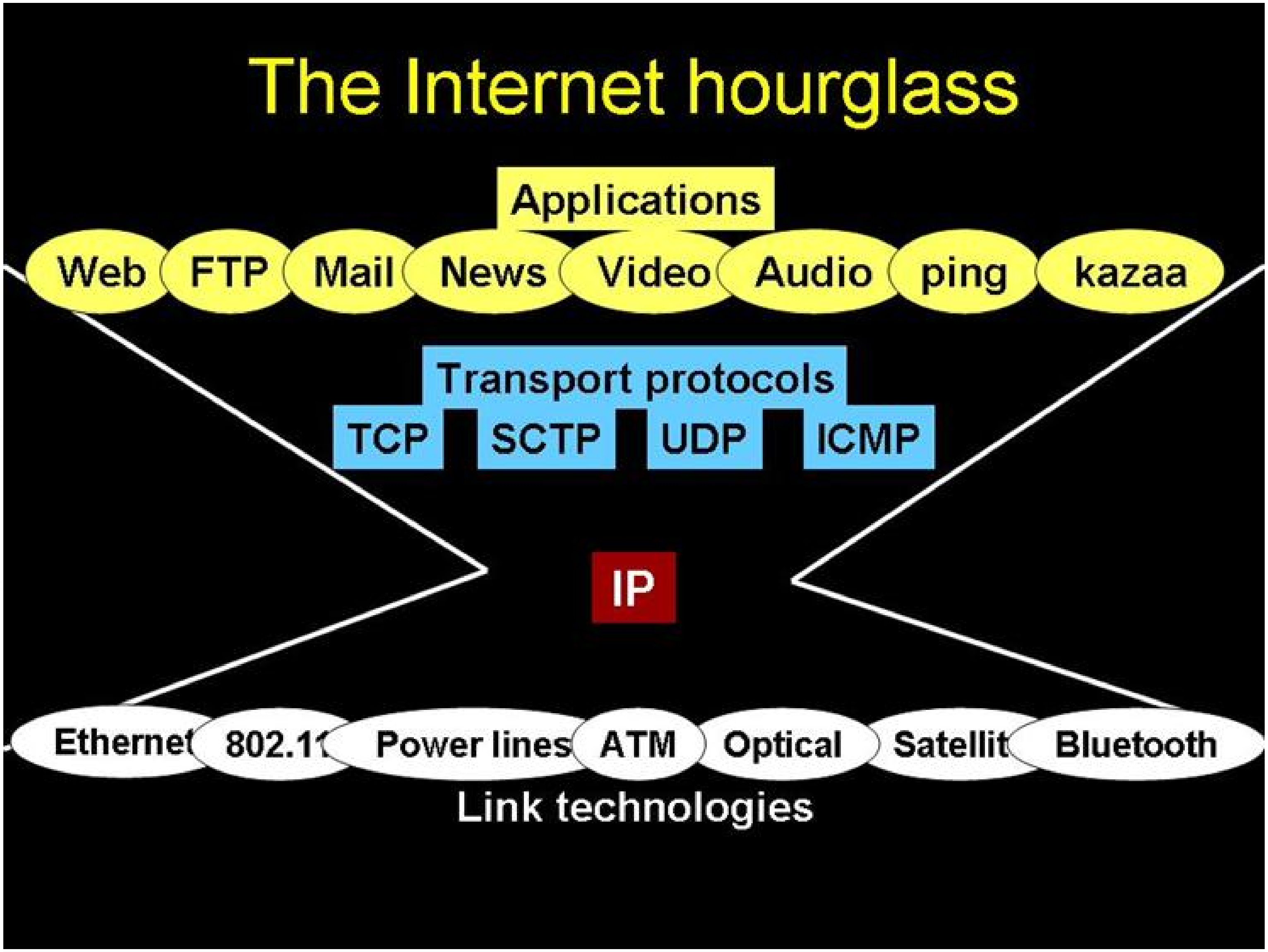
Example 1:
WWW and IP layer views of the Internet

- Each WWW link virtually connects two IP addresses.
- Those two IP nodes are typically far apart in the underlying IP topology, so the virtual connection is realized as a multihop path along IP routers.
- (Of course the IP network is then mapped onto the physical layer of optical cables and routers.)

AS
(BGP routes)

TraceRoute

Router level



(picture from David Alderson)

Example 2:
Transportation networks

Up until now separate studies of:

1. Physical topology (of roads)

2. Real-life traffic patterns

Want a comprehensive view analyzing them both together.

The “multilayer” formalism

Consider two different networks:

- $G^{\phi} = (V^{\phi}, E^{\phi})$; the physical graph.
- $G^{\lambda} = (V^{\lambda}, E^{\lambda})$; the logical/application-layer graph.

Assume both sets of nodes identical, $V^{\phi} = V^{\lambda}$.

- The *logical* edges are made of a collection of *physical* edges. $M(e^{\lambda})$ is a mapping of logical onto physical edges.

The load on a node

- Load on node i, $l(i)$, is the sum of the weights of all logical edges whose paths traverse i.
- E.g., in a transportation network $l(i)$ is the total amount of traffic that flows through node i.

Here G^ϕ is physical connection (e.g., road, rail line), G^λ "logical" is the actual flow.

Application

Study three transportation systems:

1. Mass transit system of Warsaw Poland (city).
2. Rail network of Switzerland (country).
3. Rail network of major trains in the EU (continent).

- Physical graph
- Logical graph: They can estimate the real load from the timetables (some assumptions; decompose into units (one train, one bus, etc), independent of number of people).

Physical versus logical

Physical

- exponentially decaying degree distributions.
- Diameter $\sim \sqrt{N}$

Logical

- Right skewed degree distributions
- Shorter shortest paths (e.g, number of transfers).

Load

They can estimate the real load from the timetables (some assumptions; decompose into units (one train, one bus, etc), independent of number of people).

Two typical load estimators:

1. The node degree of the physical network.

2. Betweenness of the physical network.

(Note, these estimators are the ones currently in use in almost all cases: 1) Resilience of networks to edge removal, 2) Modeling cascading failures, etc.....)

Findings

[M. Kurant and P. Thiran, “Layered Complex Networks”, Phys Rev Lett. 89, 2006.]

- All three estimators 1) real load, 2) degree, 3) betweenness differ from one-another.
- Using the two-layer view can see the logical graphs may have radically different properties than the physical graphs.
- May lead to reexamination of network robustness (previous studies on Internet, power grid, etc, based on physical layer).

Additional References

Follow up

- M. Kurant, P. Thiran and P. Hagmann "Error and Attack Tolerance of Layered Complex Networks", Phys. Rev. E, Vol. 76, 2007.

  Multilayer systems much more vulnerable to error and intentional attack than they seem from single-layer perspective.

- M. Kurant and P. Thiran "Extraction and analysis of traffic and topologies of transportation networks", Phys. Rev. E, Vol. 74, 2006.

David Aldous

- Flows through random networks
- "Spatial Transportation Networks with Transfer Costs: Asymptotic Optimality of Hub and Spoke Models"

More flows

- Flows of material goods, self-organization:
  Helbing et al.

- Jamming and flow (phase transitions):
  Nishinari, Liu, Chayes, Zechina.