**MAE 298, Network Theory and Applications**
Winter 2008
**Problem Set # 1, Due Jan 28th**

## Problem 1: Power Law Degree Distributions

Consider the power law distribution $p(k) = Ak^{-\gamma}$, with support (i.e., defined from) $k = 1$ to $k \to \infty$. In the steps below, you can either treat the $k$'s as discrete (which relies on Riemann Zeta function, $RZ(\gamma)$, and assume $\gamma$ is an integer) or use a continuum approximation (as we did in class):

$$\sum_{k=1}^{\infty} p_k \approx \int_{k=1}^{\infty} p_k dk.$$

a) Show that we must have $\gamma > 1$ for this to be a properly defined probability distribution function (pdf). Recall a pdf must have two properties: 1) $p(k) \geq 0$ for all $k$, and 2) it must be normalized.

b) Solve for the normalization constant $A$.

c) Show that if $1 < \gamma \leq 2$, the average value $\langle k \rangle$ diverges.

d) Show that if $2 < \gamma \leq 3$, the average is finite, but the standard deviation, $\sigma^2$, diverges. The best way to do this is to realize $\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2$. Derivation:

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (k_i - \langle k \rangle)^2 = \frac{1}{N} \left[ \sum_i (k_i)^2 - 2 \sum_i k_i \langle k \rangle + \sum_i \langle k \rangle^2 \right] = \left\langle k^2 \right\rangle - \langle k \rangle^2 .$$

e) Plot $p(k) = Ak^{-\gamma}$, for $k = 1$ to $k = 100,000$ for $\gamma = 3$ (and properly normalize). Use matlab, R, or pen and paper, etc (but make sure to label axes clearly with values).

f) Using the result in part (e), what is the probability for observing a node of degree $k = 1$? (If you used a continuum approximation this will be tricky, but give it your best guess.)

g) In a finite network with $N$ nodes, what is the largest possible value of degree, $k_{\max}$, that can ever be observed? So can we ever have $\langle k \rangle \to \infty$ in a finite network?

**Problem 2: Chose either Option 1 or Option 2**

**Option 1:** Consider the Barabasi-Albert (BA) model we analyzed in Lecture 3 using rate equations. Do the same analysis for the BA model with $m = 1$, but with uniform attachment (rather than preferential attachment). In other words the probability for an incoming node to attach to a node of degree $j$, $P_{ij} = 1/\sum_j d_j$. Your end result will be a simple formula for the resulting degree distribution, $p_k$. Is $p_k$ still a power law?

**Option 2:** Choose a paper of interest to you, which overlaps with the class content in some way, and write a one page (single spaced) review/critique. Include discussion of how the paper ties into network theory or how the issues discussed in the paper might benefit from network considerations.