

THE LOCALIZED EIGENSTRAIN METHOD FOR DETERMINATION OF TRIAXIAL RESIDUAL STRESS IN WELDS

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ABSTRACT

This paper discusses the localized eigenstrain method, a new technique for the determination of triaxial residual stresses deep within long welded joints. The technique follows from the “Inherent Strain Method” developed by Ueda, which uses a combined experimental and analytical approach to determine the source of residual stress (i.e., “eigenstrain” or, equivalently, “inherent strain”) by sectioning and strain measurement. Once the eigenstrain field is found, it is used to deduce residual stress in the original body prior to sectioning. The main advantage of this method is that residual stress is estimated within the entire welded joint. On the other hand, the localized technique focuses on finding residual stress only in the bead region of the joint, often a critical region with respect to fracture and fatigue. Because the method focuses only on near-bead residual stress, the required experimental effort is greatly reduced in comparison with Ueda’s original formulation. In the paper, we begin by presenting the eigenstrain method in general and then describe localization of the technique. A model problem is then developed and used to investigate the accuracy of the technique through numerical modeling. Finally, the advantages and drawbacks of this new method are discussed.

NOMENCLATURE

ϵ^*	eigenstrain tensor
ϵ^*_{ij}	individual component of eigenstrain
σ	stress tensor
σ_{ij}	individual component of stress
ξ^*	assembled vector of eigenstrain interpolation parameters
σ	assembled vector of measured stress changes
M	linear system relating σ to ξ^*
W, T, D	width, thickness, and depth of welded sample
B	size of the region of interest
ξ	non-dimensional measure within the chunk geometry

INTRODUCTION

Ueda has proposed a general method for determining triaxial residual stress, the “Inherent Strain Method” (Ueda, 1975). A distinctive characteristic of this class of procedures is that residual stress is found through estimation of the *source of residual stress* within an object. Experts on continuum mechanics and elasticity, including Timoshenko and Goodier (1970) and Mura (1987), acknowledge that residual stress is the result of some inelastic strain field which does not satisfy compatibility. This strain field is present due to mechanical and thermal processes which the body has undergone. Ueda refers to the inelastic, non-compatible strain as “inherent strain”, while we will adopt Mura’s terminology, by calling it “eigenstrain”. For a welded joint, the eigenstrain field is the combination of thermal, transformation, and plastic strains which are the net result of the welding process.

Although residual stress is caused by eigenstrain, it is also a function of the geometry of the body in which it is imposed. For example, imagine a long welded plate. If a sample is removed from the plate that is short along the weld-length, stress will be released in removing the sample. The stress has changed, but the eigenstrain within the removed sample remains the same, assuming that the cutting process results only in elastic deformation of the sample. The eigenstrain method is a form of destructive sectioning, as strains released during geometry changes are used to deduce the underlying eigenstrain distribution. The eigenstrain distribution is complicated, however, since it represents the spatial variation of a tensor quantity. If the eigenstrain can be found, it can then be used to estimate residual stress in the original sample, prior to cutting, and also in the structure from which the sample was removed.

Application of the eigenstrain approach to residual stress in continuously welded joints was presented by Ueda, et al. (1985) and further studied in a recent paper (Hill and Nelson, 1995). Consider a long, continuously welded plate with directions which correspond to the weld bead as shown in Fig. 1. These consist of the transverse,

perpendicular, and longitudinal directions relative to the direction of welding. We have chosen corresponding coordinates x , y , and z , respectively. The assumption of continuous welding allows consideration of an eigenstrain field that is dependent on the transverse and perpendicular coordinates, while independent of the longitudinal coordinate. The basis for this assumption is that each plane in the weld cross-section, like the two shaded in Figure 2, is thought to experience the same thermal and mechanical processes during a continuous weld pass. This assumption holds neither in the thermal nor the mechanical sense near the ends of the joint, but the assumption may be reasonable in the remainder of the joint, as indicated by experimental evidence presented by Hill and Nelson (1996).

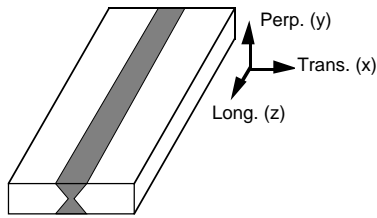


Figure 1 – Directions relative to a weld.

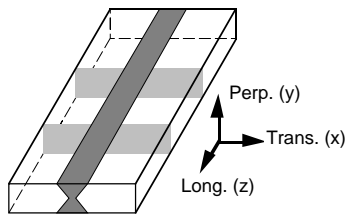


Figure 2 – Two planes which have the same eigenstrain distribution.

An illustration of Ueda's technique is shown in Figure 3. A sample of welded plate, obtained from the structure of interest, is instrumented with an array of strain gages on one of its faces normal to the z -axis (shaded in the figure). Two strain-relaxation measurements are then performed, one from block to thin slice and the other from slice to small pieces (dice), each containing a strain gage. If they are cut small enough, the dice can be assumed to be stress-free. With this assumption, the two strain-relaxation measurements, and elastic plane stress constitutive relations, experimental estimates of stress on the free-surface of the slice (σ_s) and the free-surface of the block (σ_b) are obtained. The eigenstrain technique then uses these stress estimates to determine the eigenstrain causing the two free-surface stress fields. Once the eigenstrain is found, it can be used to estimate residual stress which existed in the entire specimen geometry prior to sectioning

The determination of eigenstrain components from measured changes in strain involves solution of a linear system found by repetitive finite element method calculations, as described by Hill and Nelson (1995). For a given sectioning operation, the linear system relates measured changes in stress at specific points on the object to parameters of an eigenstrain interpolation. This linear system is very much like the stiffness matrix of finite element analysis (FEA) which

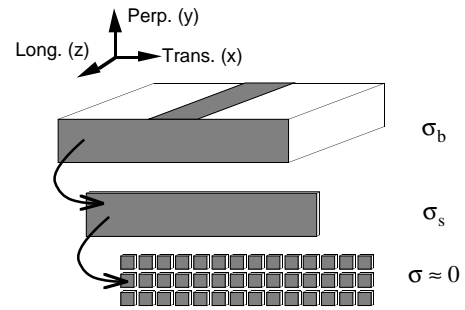


Figure 3 – Pictorial representation of the slice-and-dice method proposed by Ueda (1985).

relates internal forces to internal displacements. Just as in FEA, the formulation of the linear system depends on the type of interpolation chosen (i.e., the "shape functions") and the organization of the interpolation parameters (i.e., "assembly"). In all of the work reported here, we choose to approximate the distribution of each component of eigenstrain using linear interpolation in spatial coordinates. Once the interpolation scheme and an arbitrary system organization (row and column order) are adopted, a linear system can be formed relating stress to eigenstrain,

$$\sigma = \mathbf{M} \cdot \epsilon^* \quad [1]$$

Again, this linear system is formulated for specified locations of stress measurement and a specific sectioning process. Given the measured stress changes, σ , accompanying the sectioning process, the eigenstrain parameters, ϵ^* , can be found by inversion of Equation [1]. Usually, more than one system following the general form of Equation [1] is used to determine the complete eigenstrain tensor.

Returning to the sectioning plan shown in Figure 3, the slice is assumed to be cut thinly enough to be in a state of plane stress. The residual stress present in the slice, then, is only a function of the x - y distribution of the eigenstrain components lying in the x - y plane, which we denote $\epsilon^*_c = \{ \epsilon^*_{xx}, \epsilon^*_{yy}, \epsilon^*_{xy} \}$. The approximate distribution of these components is found using the experimental estimate of stress in the slice in an eigenstrain linear system following the general form of Equation [1].

The x - y distribution of the longitudinal eigenstrain component, ϵ^*_{zz} , is found using the estimated distribution of the planar eigenstrain components (ϵ^*_c) with the free-surface block stress estimates obtained from relaxation (σ_b). To perform this task, an elastic finite element model of the block geometry is made and the estimated planar eigenstrain components are imposed in the model. This computation provides stresses caused by ϵ^*_c on the free-surface of the block, which we may denote σ_b^c . The difference between σ_b and σ_b^c is due to the unknown distribution of ϵ^*_{zz} , which is found by formulating a linear system following the general form of Equation [1]. (The two additional components of the eigenstrain tensor (ϵ^*_{yz} and ϵ^*_{zx}) are assumed to be zero. This assumption is often made because non-zero ϵ^*_{yz} and ϵ^*_{zx} would cause asymmetrical stresses to arise in the welded joint which are contrary to empirical observation (Ueda, et al., 1985)).

Ueda's method, then, allows determination of the x - y distribution of the entire eigenstrain tensor and assumes the distribution is independent of z . Since the total eigenstrain field can be determined, residual stress which existed prior to sectioning can be found in the original specimen geometry. This is accomplished by solving the differential equations of equilibrium within the original specimen geometry, accounting for the presence of the eigenstrain field. In most cases, this can be performed by employing an elastic finite element formulation which will allow for the solution of eigenstrain-induced (residual) stress in a way that accommodates a general specimen shape and an arbitrary eigenstrain distribution (Hill, 1995).

A major drawback of Ueda's method is that a daunting number of strain-relaxation measurements must be conducted using sectioning and strain gage instrumentation. Recently, the authors have developed a localization of Ueda's approach which allows stress to be determined only in close proximity to the weld bead (Hill, 1996b). Limitation to this region is reasonable from the standpoint of failure assessment, as weld defects and large residual stresses are likely to exist together only in or very near the weld bead. Since the transverse extent of the weld bead is often on the order of the thickness of the welded plate, the localized technique was developed to find residual stress within the hexahedral region shaded in Figure 4.

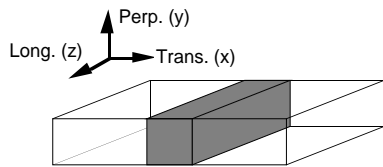


Figure 4 – Region in a welded block where residual stress is determined by the localized eigenstrain method.

DESCRIPTION OF THE LOCALIZED EIGENSTRAIN METHOD

Localization of the eigenstrain method for a continuous weld is based on a modification of the sectioning technique proposed by Ueda and is shown schematically in Figure 5. This experimental procedure differs from that shown in Figure 3 in two ways. First, only the bead region on the face of the welded sample, shown shaded in the figure, is instrumented with strain gages. Secondly, the sectioning process includes an intermediate "chunk" geometry between slice and dice, created by removing the region of interest from the slice. This modified sectioning process allows for separation of the eigenstrain field into two portions, one lying inside and one outside of the region of interest.

The localized eigenstrain method is most easily described with a specific geometry, sectioning plan, and strain measurement scheme in mind. Assume that a block of material has been removed from a welded joint and has dimensions shown in Figure 6(a). A region of interest in the xy -plane of this block is identified, shown shaded in the figure. The free-surface of this region is instrumented as shown in Figure 7. Following instrumentation, the block is sectioned as shown schematically in Figure 5 with dimensions of the slice and chunk

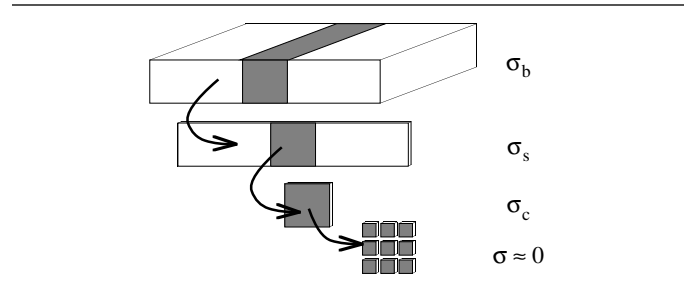


Figure 5 – Pictorial representation of the localized eigenstrain technique for continuous welds.

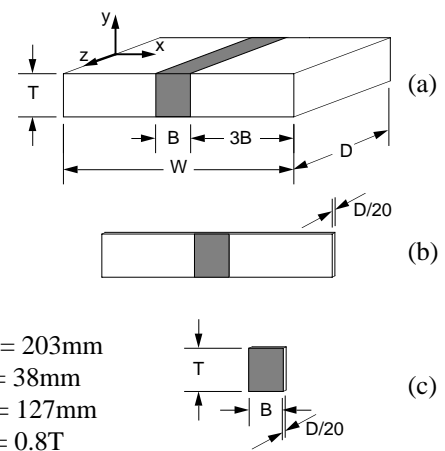


Figure 6 – (a) Block of material removed from a welded joint, (b) slice removed from the block, and (c) chunk removed from the slice. The chunk would further be sectioned into dice.

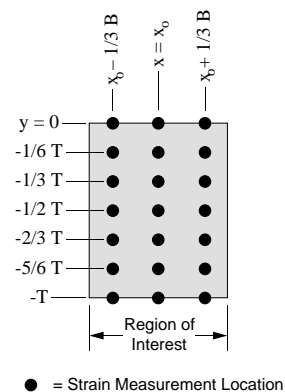


Figure 7 – Measurement locations within the region of interest where three-element strain gage rosettes would be placed.

shown in Figure 6(b) and (c). Strain changes are measured that accompany removal of the slice from the block, removal of the chunk from the slice, and finally cutting the chunk into dice. Assuming that the dice are small enough relative to the spatial gradients of eigenstrain, the dice will be stress-free. Using strain relaxation data with elastic stress-strain relations for plane stress, residual stress can then be computed at the free-surface measurement sites on the chunk, slice, and block geometries. The localized eigenstrain method subsequently uses these reduced stress data to estimate the eigenstrain distribution which, in turn, allows stress to be computed at any point in the region of interest, including points remote from the free-surface.

Division of eigenstrain into ϵ^*_A and ϵ^*_B

One of the crucial steps in assuring success of the localized eigenstrain method is separation of the eigenstrain into two parts. The first part, ϵ^*_B , is found from stress in the chunk, and the second part, ϵ^*_A , from stress in the slice. This second part of the eigenstrain is further divided into two distinct parts. The first is eigenstrain which lies within the region of the chunk, but does not cause stress once the chunk is cut free from the slice (e.g., a uniform strain restricted only to the chunk region). The second part of ϵ^*_A is an approximate representation for eigenstrain which lies outside of the chunk region but causes stress inside the chunk region. To correctly account for this division, the x -interpolation for each portion of the eigenstrain has been carefully developed (Hill, 1996b). The y -interpolation, on the other hand, is a simple linear interpolation with seven equally spaced nodes lying at the strain measurement locations (i.e., $y = \{ 0, -T/6, \dots, -T \}$, see Figure 7).

Scheme for interpolation of ϵ^*_B

Functions used to interpolate ϵ^*_B in the x -direction are shown in Figure 8. These functions are the result of two important considerations. First, eigenstrain distributions which would cause stress-free deformation in the chunk are explicitly excluded by proper choice of the interpolation scheme. For each particular component of eigenstrain, this amounts to assuring that the compatibility relations of elasticity cannot be satisfied by any combination of the interpolation functions. If such a zero-stress producing combination were present, the inversion of Equation [1] would be impossible. Second, spacing between interpolation nodes for a particular component of eigenstrain is constant. It was found that even spacing produces a linear system,

\underline{M} , which is as well conditioned as possible, and therefore produces superior estimates of the eigenstrain parameters. The functions shown in Figure 8 satisfy these two criteria.

Scheme for interpolation of ϵ^*_A

Again, there are two parts of the eigenstrain distribution which cause stresses in the slice and cannot be predicted by stresses in the chunk. The first part is the eigenstrain which causes stress when present in the chunk region of the slice geometry, but causes no stress once the chunk is cut free. The interpolation of this portion of ϵ^*_A varies for each component of eigenstrain, as shown in Figure 9(a) and (b). For ϵ^*_{xx} , there are no stress-free modes of eigenstrain distribution within the chunk which are not also stress-free in the slice. The second part of the eigenstrain to be determined by stress in the slice is the eigenstrain which lies outside of the chunk region. It was found, by trial and error, that the specific functions chosen to interpolate this part of the eigenstrain do not alter the results of the stress approximation *within the region of interest*. Therefore, the simple sawtooth functions shown in Figure 9(c) are used to interpolate each component of eigenstrain outside the region of interest.

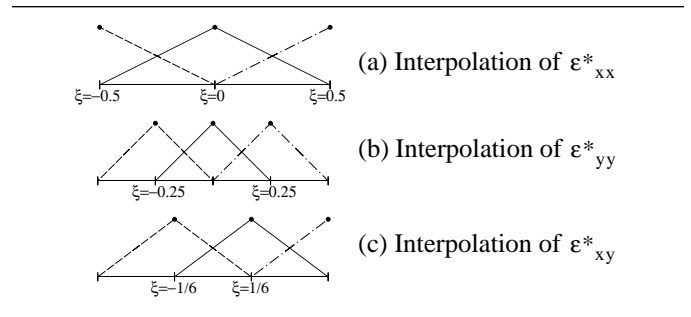


Figure 8 – Interpolation functions used to distribute ϵ^*_B in the chunk. $\xi = (x - x_0)/B$, where x_0 is the center of the chunk and B is the width of the region of interest. Measurements taken at $\xi = -1/3, 0, 1/3$.

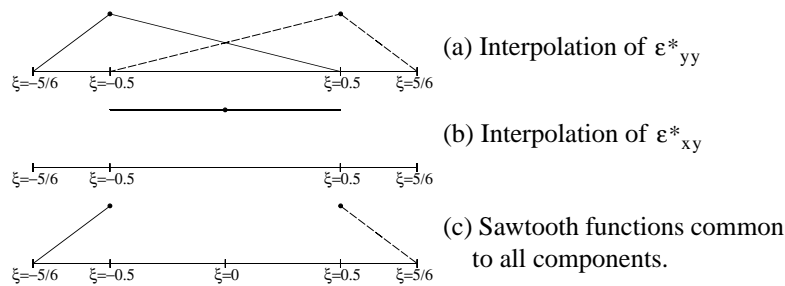


Figure 9 – Interpolation functions used to distribute eigenstrain determined from stress in the slice, ϵ^*_A . $\xi = (x - x_0)/B$, where x_0 is the center of the chunk.

Interpolation for longitudinal eigenstrain, ϵ_{zz}^*

In estimating the longitudinal eigenstrain component the only distinction between the chunk and the slice is a discontinuity in interpolation across the boundary of the region of interest. That is, the solution procedure for finding a parameterized distribution of ϵ_{zz}^* is the same as in the non-localized method, except for the interpolation functions used. The interpolation scheme adopted is again the result of eliminating stress-free modes in the linear system and ensuring even spacing of the interpolation nodes. The result of these considerations is an interpolation scheme for ϵ_{zz}^* which is the same as that used for ϵ_{xx}^* . The proper functions are therefore the combination of those shown in Figure 8(a) and Figure 9(c).

To find the longitudinal eigenstrain, the distribution of the planar components of eigenstrain (both ϵ_B^* and ϵ_A^*) is estimated from the experimental estimates of stresses in the chunk and slice, as described above. This eigenstrain distribution is then imposed in a finite element model of the block to find the stresses that they cause on its free-surface. These stresses are subtracted from the experimental estimates of residual stress on the block free-surface, and the differences used to determine the parameters in the ϵ_{zz}^* interpolation.

NUMERICAL SIMULATION OF THE METHOD

The accuracy of the method described above will depend greatly on the residual stress distribution being measured. Our goal in this section is to assess the accuracy with respect to one residual stress system in particular through numerical simulation of the technique.

The goal of the simulation is to find residual stress within the sample of welded plate depicted in Figure 6(a). Residual stress is produced by imposing an idealized weld-eigenstrain field in an elastic finite element model of each specimen geometry shown in Figure 6. The resulting residual stress state in each geometry is fully three-dimensional, and exists everywhere within the body. The specific eigenstrain field used to introduce residual stress is given in detail in an earlier paper (Hill and Nelson, 1995). This field was developed to produce a complicated residual stress state that *resembles* the character of thick-weld residual stress; however, this field should not be construed to *be* the residual stress state present in any real weld. Its sole purpose is to provide a basis by which to compare techniques for residual stress determination. Computed residual stresses on specific contours at the mid-length of the weld are shown in Figures 10 and 11. These results resemble published residual stress profiles for double-sided butt-welded joints (e.g., see Gunnert (1961)). Since these stresses are the result of a direct finite element computation, they are identified as “exact,” meaning that a perfect measuring technique would obtain the same results.

Linear systems are formed for determination of ϵ_B^* , ϵ_A^* , and ϵ_{zz}^* using the interpolation schemes described above. Details on the general method used to form an eigenstrain linear system can be found in (Hill, 1995). These systems are then used with stress on the free-surfaces of the chunk, slice, and block computed by FEA. That is, the FEA results on the free-surface are used as input to the localized eigenstrain method in place of actual experimental measurements. Parameters determined from solution of these systems are used to

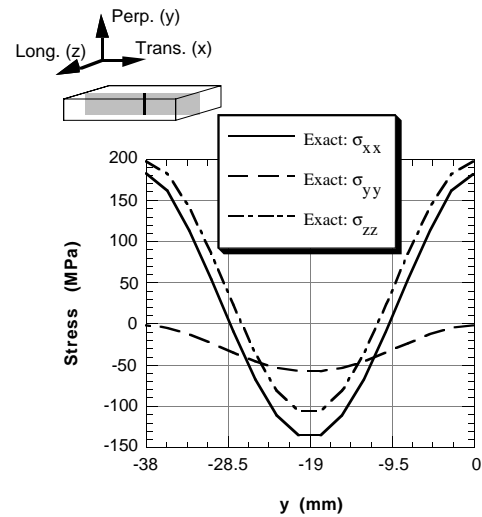


Figure 10 – Residual stresses at the center of the sample, through the thickness of the plate.

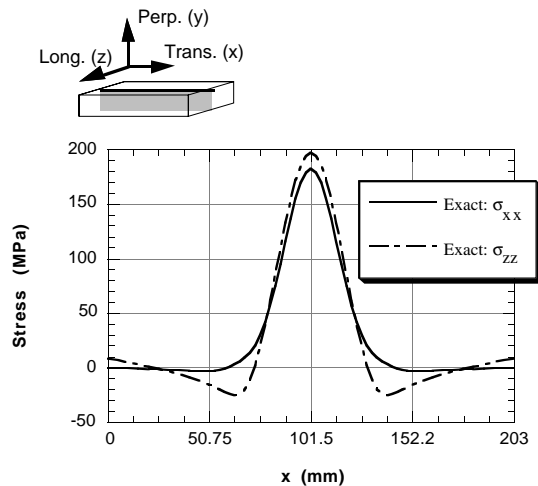


Figure 11 – Residual stress at the center of the weld length, across the top surface.

interpolate the total estimated eigenstrain distribution in a finite element model of the block geometry, and residual stress estimates in the block interior are obtained. Stresses estimated at the weld midlength are compared to FEA results in Figure 12. Good agreement between the finite element computation and the eigenstrain estimation is obtained. The largest errors present are in σ_{zz} near the surface of the joint.

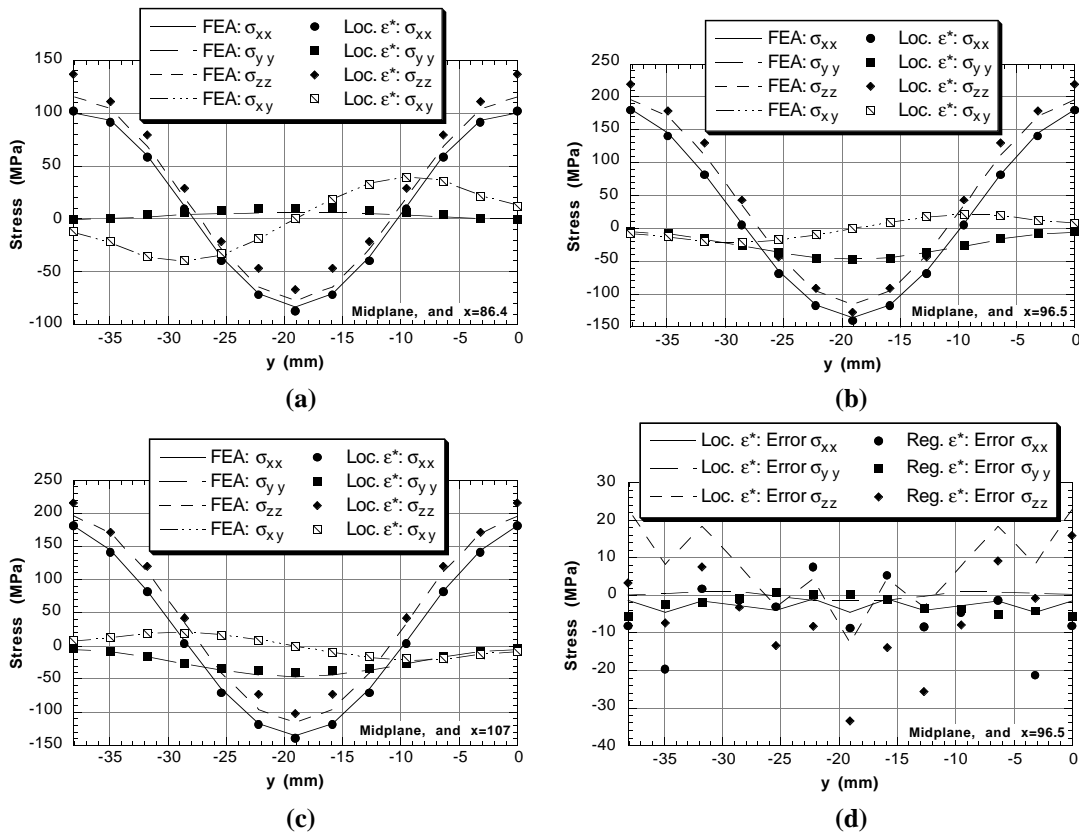


Figure 12 – (a), (b), (c): Stress at the midplane of the block induced by the model eigenstrain function as computed by FEA, compared to those estimated by the localized eigenstrain method. Each plot is for a different x -location, as indicated at the bottom-right. (d) Error in mid-plane stress estimates for the localized eigenstrain technique and the "regular" technique proposed by Ueda (adapted from (Hill and Nelson, 1995)) on the line $x = 96.5$ mm.

DISCUSSION

Numerical simulation indicates that the localized eigenstrain method can be used to estimate residual stresses at the midplane of the block to a good degree of accuracy. The maximum difference between estimated and exact stress is smaller than that obtained using the "non-localized" eigenstrain method, as shown in Figure 12(d). The improved accuracy is the result of the improved shape functions used to interpolate the eigenstrain. In developing these functions, an effort was made to minimize the number of eigenstrain components which are lost to modes of stress-free deformation during the inversion of Equation [1]. Overall, the difference between estimated and exact stress for either method is fairly small in relation to the levels of stress present in the block, as shown in Figure 12(a)–(c).

The development of the localized eigenstrain technique adds to the ability of the technique developed by Ueda. The localized method has the advantages of Ueda's method, while being easier to implement for determination of stresses near a weld bead. The experimental effort required for the two techniques differs greatly. Implementation of Ueda's technique, as described by Hill and Nelson (1995), would

require 140 three-element strain gage rosettes. The localized technique, on the other hand, would require only 21. This is a large reduction in effort, which makes use of the eigenstrain technique much more practical. Not only is the experimental effort reduced, but the computational burden as well, since one finite element solution must be obtained for each eigenstrain parameter in the interpolation system. As described by Hill and Nelson (1995), Ueda's technique had 500 such parameters while the method presented here has only 147. For these reasons, the localized eigenstrain method adds ease of execution to the previously existing method developed by Ueda.

CONCLUSION

A localized version of Ueda's inherent strain (or, eigenstrain) method has been developed to enable determination of weld residual stresses by sectioning with a considerable reduction in experimental and computational efforts required. The new method retains the ability of the eigenstrain approach to estimate triaxial residual stress through the thickness of the welded joint but obtains results only in the bead region.

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