A Simplified Eigenstrain Approach for Determination of Sub-surface Triaxial Residual Stress in Welds

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Abstract

This paper discusses the localized eigenstrain method, a new technique for the determination of triaxial residual stresses deep within long welded joints. The technique follows from the "Inherent Strain Method" developed by Ueda, which uses a combined experimental and analytical approach to determine the source of residual stress (i.e., "eigenstrain" or, equivalently, "inherent strain") by sectioning and strain measurement. Once the eigenstrain field is found, it is used to deduce residual stress in the original body prior to sectioning. The main advantage of this method is that residual stress is estimated within the entire welded joint. On the other hand, the localized technique focuses on finding residual stress only in the bead region of the joint, often a critical region with respect to fracture and fatigue. Because the method focuses only on near-bead residual stress, the required experimental effort is greatly reduced in comparison with Ueda's original formulation. In the paper, we begin by presenting the eigenstrain method in general and then describe localization of the technique. A model problem is then developed and used to investigate the accuracy of the technique through numerical modeling. Finally, the advantages and drawbacks of this new method are discussed.

Nomenclature

ε^*	eigenstrain tensor
ε^*_{ij}	individual component of eigenstrain
σ	stress tensor
σ_{ij}	individual component of stress
$\underset{\sim}{\varepsilon}^*$	assembled vector of eigenstrain interpolation parameters
$\underset{\sim}{\sigma}$	assembled vector of stress changes
$\underset{\sim}{M}$	linear system relating stress to eigenstrain
W, T, D	width, thickness, and depth of welded sample
B	size of the region of interest
ξ	non-dimensional measure within the chunk geometry

1 Introduction

Ueda has proposed a general method for determining triaxial residual stress, the "Inherent Strain Method" (Ueda, 1975). A distinctive characteristic of this class of procedures is that residual stress is found through estimation of the *source of residual stress* within an object. Experts on continuum mechanics and elasticity, including Timoshenko and Goodier (1970) and Mura (1987), acknowledge that residual stress is the result of some inelastic strain field which does not satisfy compatibility. This strain field is present due to mechanical and thermal processes which the body has undergone. Ueda refers to the inelastic, non-compatible strain as "inherent strain", while we will adopt Mura's terminology, by calling it "eigenstrain". For a welded joint, the eigenstrain field is the combination of thermal, transformation, and plastic strains which are the net result of the welding process.

Although residual stress is caused by eigenstrain, it is also a function of the geometry of the body in which it is imposed. For example, imagine a long welded plate. If a sample is removed from the plate that is short along the weld-length, stress will be released in removing the sample. The stress has changed, but the eigenstrain within the removed sample remains the same, assuming that the cutting process results only in elastic deformation of the sample. The eigenstrain method is a form of destructive sectioning, as strain released during geometry changes is used to deduce the underlying eigenstrain distribution. The eigenstrain distribution is complicated, however, since it represents the spatial variation of a tensor quantity. If the eigenstrain can be found, it can then be used to estimate residual stress in the original sample, prior to cutting, and also in the structure from which the sample was removed.

Application of the eigenstrain approach to residual stress in continuously welded joints was presented by Ueda, et al. (1985) and further studied in a recent paper (Hill and Nelson, 1995). Consider a long, continuously welded plate with directions which correspond to the weld bead as shown in Figure 1. These consist of the transverse, perpendicular, and longitudinal directions relative to the direction of welding. We have chosen corresponding coordinates x, y, and z, respectively. The assumption of continuous welding allows consideration of an eigenstrain field that is dependent on the transverse and perpendicular coordinates, while independent of the longitudinal coordinate. The basis for this assumption is that each plane in the weld cross-section, like the two shaded in Figure 2, is thought to experience the same thermal and mechanical processes during a continuous weld pass. This assumption holds neither in the thermal nor the mechanical sense near the ends of the joint, but the assumption may be reasonable in the remainder of the joint, as indicated by experimental evidence presented by Hill and Nelson (1996).

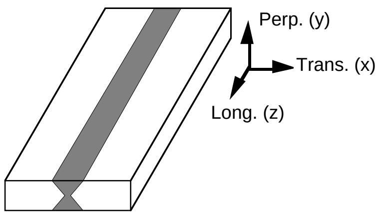

Figure 1 – Directions relative to a weld.

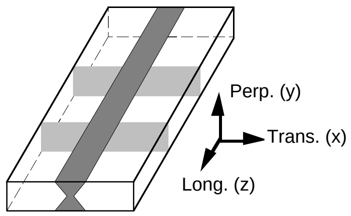

Figure 2 – Two planes which have the same eigenstrain distribution.

An illustration of Ueda's technique is shown in Figure 3. A sample of welded plate, obtained from the structure of interest, is instrumented with an array of strain gages on one of its faces normal to the z-axis (shaded in the figure). Two strain-relaxation measurements are then performed, one from block to thin slice and the other from slice to small pieces (dice), each containing a strain gage. Since the slice is in plane stress, slice-to-dice data allow determination of the x-y distribution of the eigenstrain components in the transverse-perpendicular plane (ε^*_{xx}, ε^*_{yy}, and ε^*_{xy}). Using these results with the block-to-slice relaxation data allows determination of the eigenstrain component associated with the longitudinal direction (ε^*_{zz}). The two additional components of eigenstrain (ε^*_{yz} and ε^*_{zx}) are assumed to be zero because they would cause asymmetrical stresses to arise in the welded joint which is contrary to empirical observation (Ueda, et al., 1985). Ueda's method, then, allows determination of the x-y

distribution of the entire eigenstrain tensor and assumes the distribution is independent of *z*. Since the total eigenstrain field is known, residual stress can be found in the original specimen geometry, prior to sectioning, or in the structural geometry prior to specimen removal by imposing the eigenstrain field in an elastic finite element model (Hill, 1995).

The determination of eigenstrain components from measured changes in strain involves solution of a linear system found by repetitive finite element method calculations, as described by Hill and Nelson (1995). For a given sectioning operation, the linear system relates measured changes in stress at specific points on the object to parameters of an eigenstrain interpolation. This linear system is very much like the stiffness matrix of finite element analysis (FEA) which relates internal forces to internal displacements. Just as in FEA, the formulation of the linear system depends on the type of interpolation chosen (i.e., the "shape functions") and the organization of the interpolation parameters (i.e., "assembly"). In all of the work reported here, we choose to approximate the distribution of each component of eigenstrain using linear interpolation in spatial coordinates. Once the interpolation scheme and an arbitrary system organization (row and column order) are adopted, a linear system can be formed relating stress to eigenstrain,

$$\underset{\sim}{\sigma} = \underset{\sim}{M} \cdot \underset{\sim}{\varepsilon}^* . \qquad [1]$$

Given measured stress components, $\underset{\sim}{\sigma}$, the eigenstrain parameters, $\underset{\sim}{\varepsilon}^*$ (akin to nodal values of displacement in FEA) can be found by inversion of Equation [1].

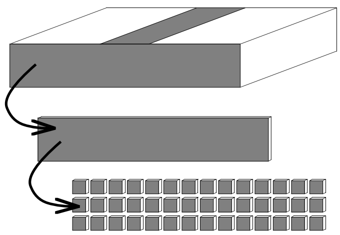

Figure 3 – Pictorial representation of the slice-and-dice method proposed by Ueda (1985).

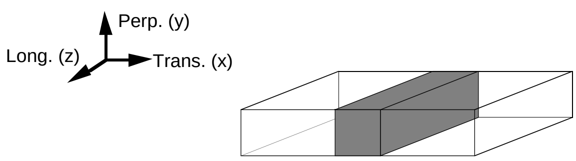

Figure 4 – Region in a welded block where residual stress is determined by the localized eigenstrain method.

A major drawback of Ueda’s method is that a daunting number of strain-relaxation measurements must be conducted using sectioning and strain gage instrumentation. Recently, the authors have developed a localization of Ueda’s approach which allows stress to be determined only in close proximity to the weld bead (Hill, 1996b). Limitation to this region is reasonable from the standpoint of failure assessment, as weld defects and large residual stresses are likely to exist together only in or very near the weld bead. Since the transverse extent of the weld bead is often on the order of the thickness of the welded plate, the localized technique was developed to find residual stress within the hexahedral region shaded in Figure 4.

2 Description of the Localized Eigenstrain Method

Localization of the eigenstrain method for a continuous weld is based on a modification of the sectioning technique proposed by Ueda and is shown schematically in Figure 5. This process differs from that shown in Figure 3 in two ways. First, only the bead region on the face of the welded sample, shown shaded in the figure, is instrumented with strain gages. Secondly, the sectioning process includes an intermediate “chunk” geometry between slice and dice, created by removing the region of interest from the slice. This modified sectioning process allows for separation of the eigenstrain field into two portions, one lying inside and one outside of the region of interest.

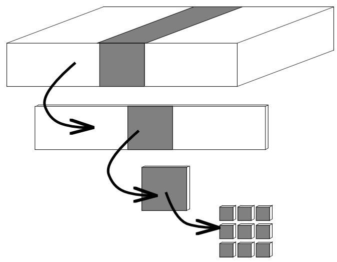

Figure 5 – Pictorial representation of the localized eigenstrain technique for continuous welds.

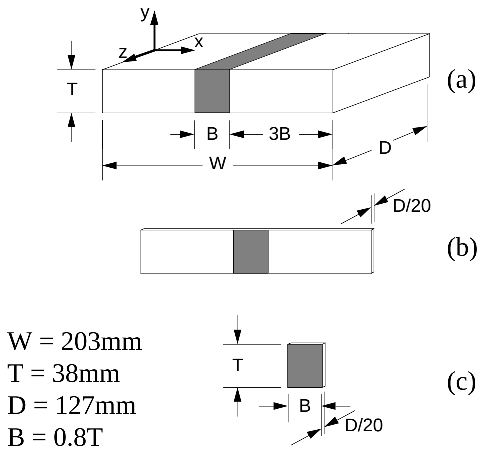

Figure 6 – (a) Block of material removed from a welded joint, (b) slice removed from the block, and (c) chunk removed from the slice. The chunk would further be sectioned into dice.

The localized eigenstrain method is most easily described with a specific geometry, sectioning plan, and strain measurement scheme in mind. Assume that a block of material has been removed from a welded joint and has dimensions shown in Figure 6(a). A region of interest in the *xy*-plane of this block is identified, shown shaded in the figure. The free surface of this region is instrumented as shown in Figure 7. Following instrumentation, the block is sectioned as shown schematically in Figure 5 with dimensions of the slice and chunk shown in Figure 6(b) and (c). Strain changes are measured that accompany removal of the slice from the block, removal of the chunk from the slice, and finally cutting the slice into dice. Assuming that the dice are small enough relative to the spatial gradients of eigenstrain, the dice will be stress-free. Using relaxation data with elastic stress-strain relations for plane stress, residual stress can then be computed at the free-surface measurement sites on the block, slice, and chunk geometries. The localized eigenstrain method then uses these reduced stress data to estimate the eigenstrain distribution which, in turn, allows stress to be computed at any point in the region of interest.

2.1 Division of eigenstrain into ε^*_A and ε^*_B

One of the crucial steps in assuring success of the localized eigenstrain method is separation of the eigenstrain into two parts. The first part, ε^*_B, is found from stress in the chunk, and the second part, ε^*_A, from stress in the slice. This second part of the eigenstrain is further divided into two distinct parts. The first is eigenstrain which lies within the region of the chunk, but does not cause stress once the chunk is cut free from the slice. The second part of ε^*_A is an approximate representation for eigenstrain which lies outside of the chunk region. To correctly account for this division, the *x*-interpolation for each portion of the eigenstrain has been carefully developed (Hill, 1996b). The *y*-interpolation, on the other hand, is a simple linear interpolation with seven equally spaced nodes.

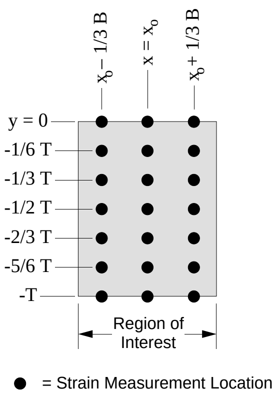

Figure 7 – Measurement locations within the region of interest where three-element strain gage rosettes would be placed.

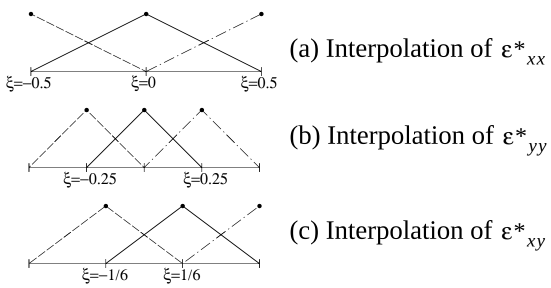

Figure 8 – Interpolation functions used to distribute ε^*_B in the chunk. $\xi = (x - x_o)/B$, where x_o is the center of the chunk and B is the width of the region of interest. Measurements taken at ξ = -1/3, 0, 1/3.

2.2 Scheme for interpolation of ε^*_B

Functions used to interpolate ε^*_B in the *x*-direction are shown in Figure 8. These functions are the result of two important considerations. First, eigenstrain distributions which would cause stress-free deformation in the chunk are explicitly excluded by proper choice of the interpolation scheme. For each particular component of eigenstrain, this amounts to assuring that the compatibility relations of elasticity cannot be satisfied by any combination of the interpolation functions. Second, spacing between interpolation nodes for a particular component is constant. It was found that even spacing produces a linear system for eigenstrain determination which is as well conditioned as possible, and therefore produces superior estimates of the eigenstrain parameters. The functions shown in Figure 8 satisfy these two criteria.

2.3 Scheme for interpolation of ε^*_A

Again, there are two parts of the eigenstrain distribution which cause stresses in the slice and cannot be predicted by stresses in the chunk. The first part is the eigenstrain which causes stress when present in the chunk region of the slice geometry, but causes no stress once the chunk is cut free. The interpolation of this portion of ε^*_A varies for each component of eigenstrain, as shown in Figure 9(a) and (b). For ε^*_{xx}, there are no stress-free modes of eigenstrain distribution which are not also stress-free in the slice. The second part of the eigenstrain to be determined by stress in the slice is the eigenstrain which lies outside of the chunk region, ε^*_A. It was found, by trial

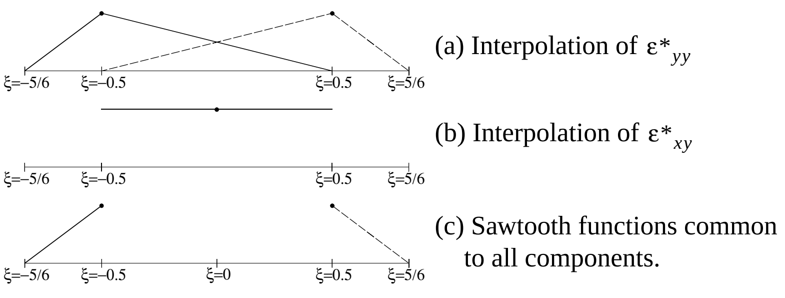

Figure 9 – Interpolation functions used to distribute eigenstrain determined from stress in the slice, ε^*_A. $\xi = (x - x_o)/B$, where x_o is the center of the chunk.

and error, that the specific functions used to interpolate this part of the eigenstrain does not alter the results of the stress approximation *within the region of interest*. Therefore, the simple sawtooth functions shown in Figure 9(c) are used to interpolate each component of eigenstrain outside the region of interest.

2.4 Interpolation for longitudinal eigenstrain, ε^*_{zz}

In estimating the longitudinal eigenstrain component, there is no distinction between the chunk and the slice, except in a discontinuity in interpolation across the boundary of the region of interest. That is, the solution procedure for finding a parameterized distribution of ε^*_{zz} is the same as in the non-localized method, except for the interpolation functions used. The proper interpolation for ε^*_{zz} is the same as that used for ε^*_{xx} as shown in Figure 8(a) and Figure 9(c).

To find the longitudinal eigenstrain, the distribution of the planar components of eigenstrain is estimated as described above, then these are imposed in a finite element model of the block to find the stresses that they cause on its free-surface. These stresses are subtracted from the experimental estimates of residual stress on the block free-surface, and the differences used to determine the parameters in the ε^*_{zz} interpolation.

3 Numerical Simulation of the Method

The accuracy of the method described above will depend greatly on the residual stress distribution being measured. Our goal in this section is to assess the accuracy with respect to one residual stress system in particular through numerical simulation of the technique.

The goal of the simulation is to find residual stress within the sample of welded plate depicted in Figure 6(a). Residual stress is produced in a finite element model of each specimen shown in Figure 6 by introduction of an eigenstrain field. The resulting residual stress state is fully three-dimensional, and exists everywhere within the body. The specific eigenstrain field used is given in detail in an earlier paper (Hill and Nelson, 1995). This field was developed to produce a complicated residual stress state that *resembles* the character of thick-weld residual stress; however, this field should not be construed to *be* the residual stress state present in any real weld. Its sole purpose is to provide a basis by which to compare techniques for residual stress determination. Residual stresses on specific contours through the weld are shown in Figures 10 and 11. These stresses are the result of a direct finite element computation and are identified as “exact,” meaning that a perfect measuring technique would obtain the same results.

The linear systems are formed for determination of ε^*_B, ε^*_A, and ε^*_{zz} using the interpolation schemes described above. These systems are then used with stress on the free-surfaces of the chunk, slice, and block computed by FEA. That is, the FEA results on the free-surface are used as input to the localized eigenstrain method. Parameters determined from solution of these systems are then used to interpolate the total eigenstrain on a model of the block geometry, and residual stress estimates within the block are obtained. Stress estimates at the weld midlength are compared to FEA results in Figure 12. Good agreement between the two sets of results is obtained. The largest errors present are in σ_{zz} near the surface of the joint.

4 Discussion

Numerical simulation indicates that the localized eigenstrain method can be used to estimate residual stresses at the midplane of the block to a good degree of accuracy. The maximum difference between estimated and exact stress is smaller than that obtained using the

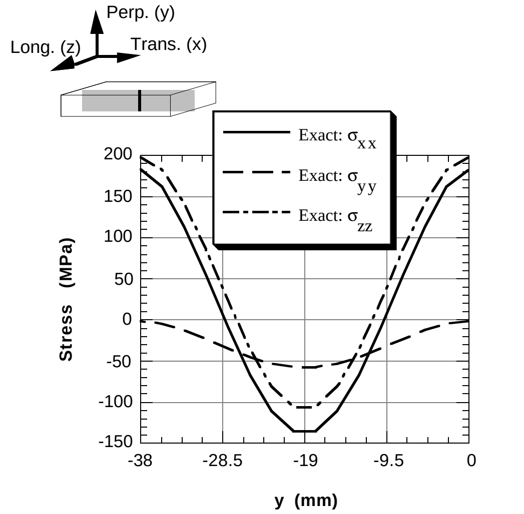

Figure 10 – Residual stresses at the center of the sample, through the thickness of the plate.

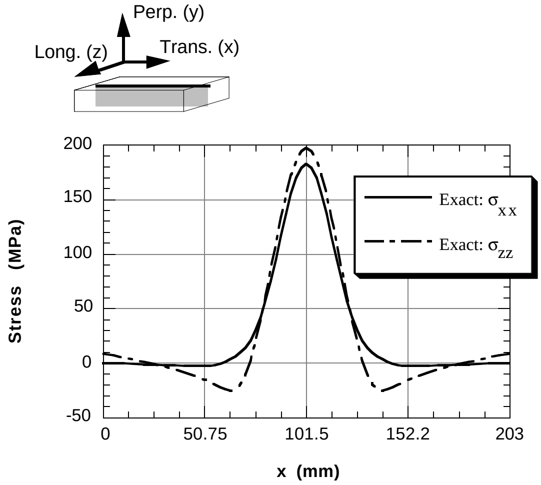

Figure 11 – Residual stress at the center of the weld length, across the top surface.

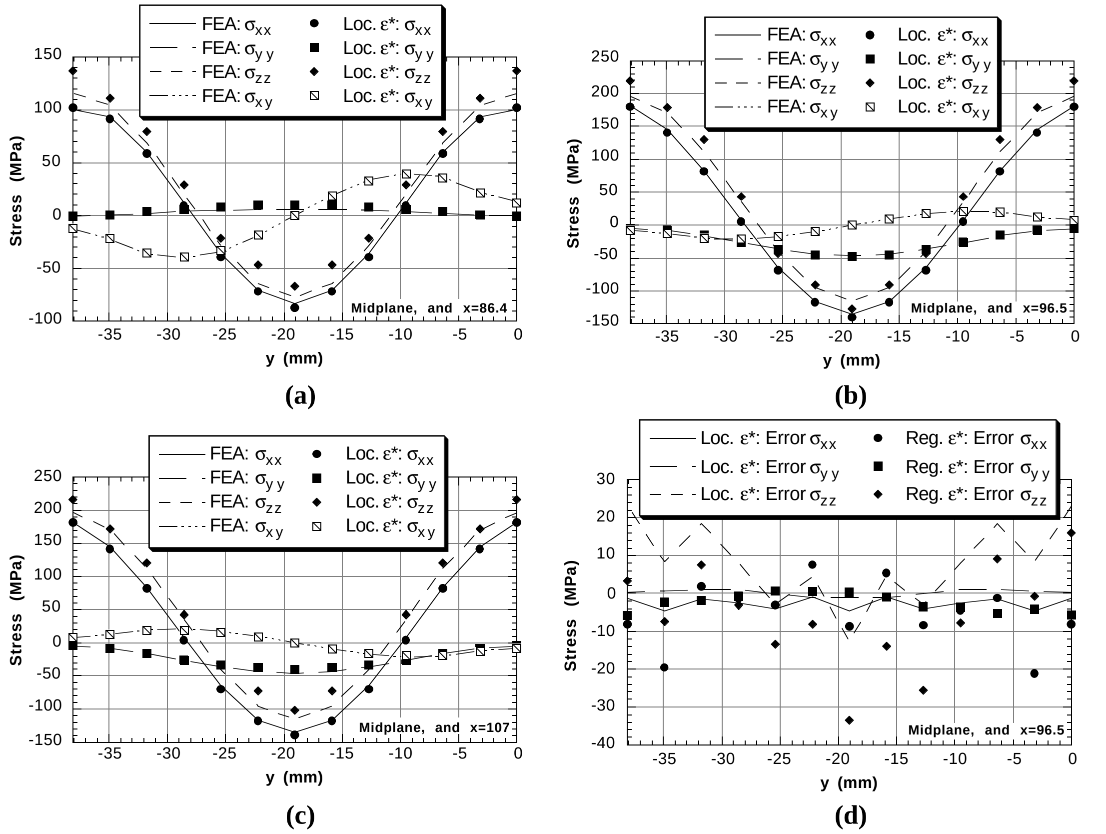

Figure 12 – (a), (b), (c): Stress at the midplane of the block induced by the model eigenstrain function as computed by FEA, compared to those estimated by the localized eigenstrain method. Each plot is for a different *x*-location, as indicated at the bottom-right.
(d) Error in mid-plane stress estimates for the localized eigenstrain technique and the "regular" technique proposed by Ueda (adapted from (Hill and Nelson, 1995)) on the line *x* = 96.5mm.

"non-localized" eigenstrain method, as shown in Figure 12(d). The improved accuracy is the result of the improved shape functions used to interpolate the eigenstrain. In developing these functions, an effort was made to minimize the number of eigenstrain components which are lost to modes of stress-free deformation during the inversion of Equation [1]. Overall, the difference between estimated and exact stress for either method is fairly small in relation to the levels of stress present in the block, as shown in Figure 12(a)–(c).

The development of the localized eigenstrain technique adds to the ability of the technique developed by Ueda. The localized method has the advantages of Ueda's method, while being easier to implement for determination of stresses near a weld bead. The experimental effort required for the two techniques differs greatly. Implementation of Ueda's technique, as described by Hill and Nelson (1995), would require 140 three-element strain gage rosettes. The localized technique, on the other hand, would require only 21. This is a large reduction in effort, which makes use of the eigenstrain technique much more practical. Not only is the experimental effort reduced, but the computational burden as well since one finite element solution must be obtained for each eigenstrain parameter in the interpolation system. As described by Hill and Nelson (1995), Ueda's technique had 500 such parameters while the method presented here has only 147. For these reasons, the localized eigenstrain method adds ease of execution to the previously existing method developed by Ueda.

5 Conclusion

A localized version of Ueda's inherent strain (or, eigenstrain) method has been developed to enable determination of weld residual stresses by sectioning with a considerable reduction in experimental and computational efforts required. The new method retains the ability of the eigenstrain approach to estimate triaxial residual stress through the thickness of the welded joint but obtains results only in the bead region.

6 References

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