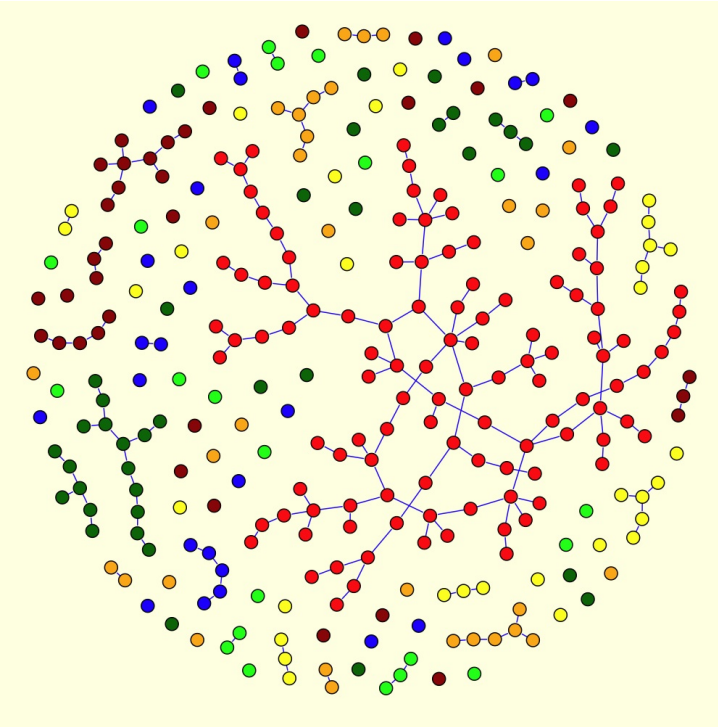
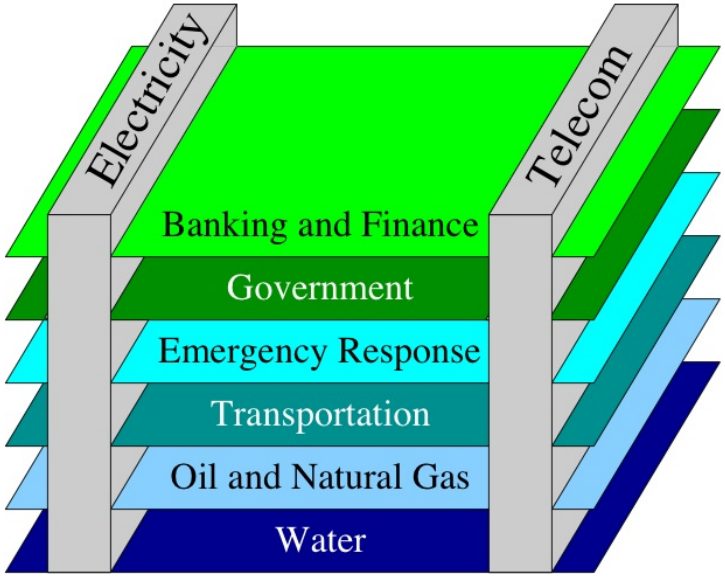
The Science of Networks:
Modeling our complex, interdependent world





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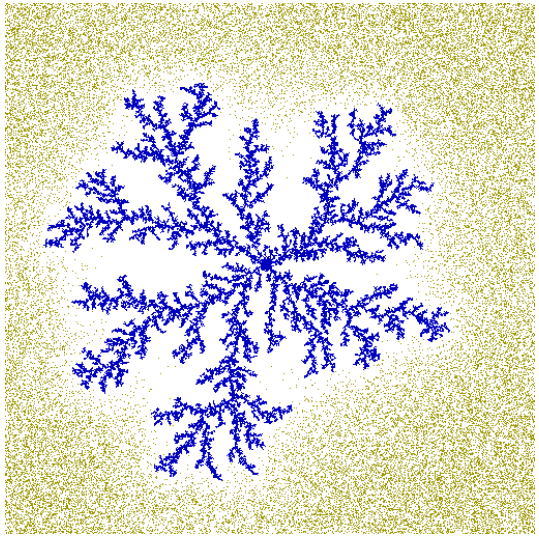
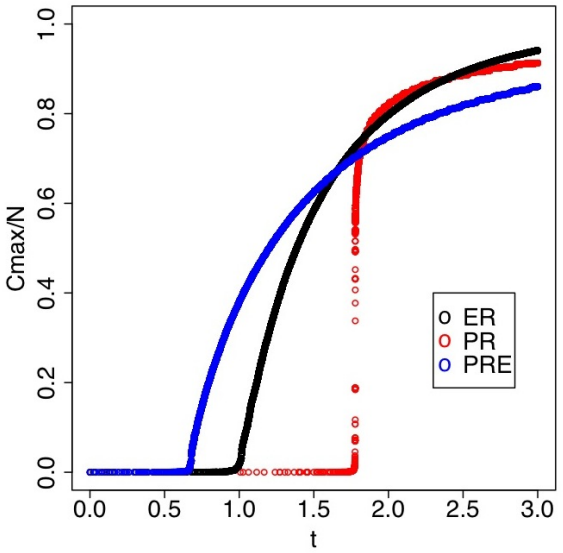
Dept of Mech. and Aero. Eng., Dept of CS
Complexity Sciences Center
External Professor, **Santa Fe Institute**

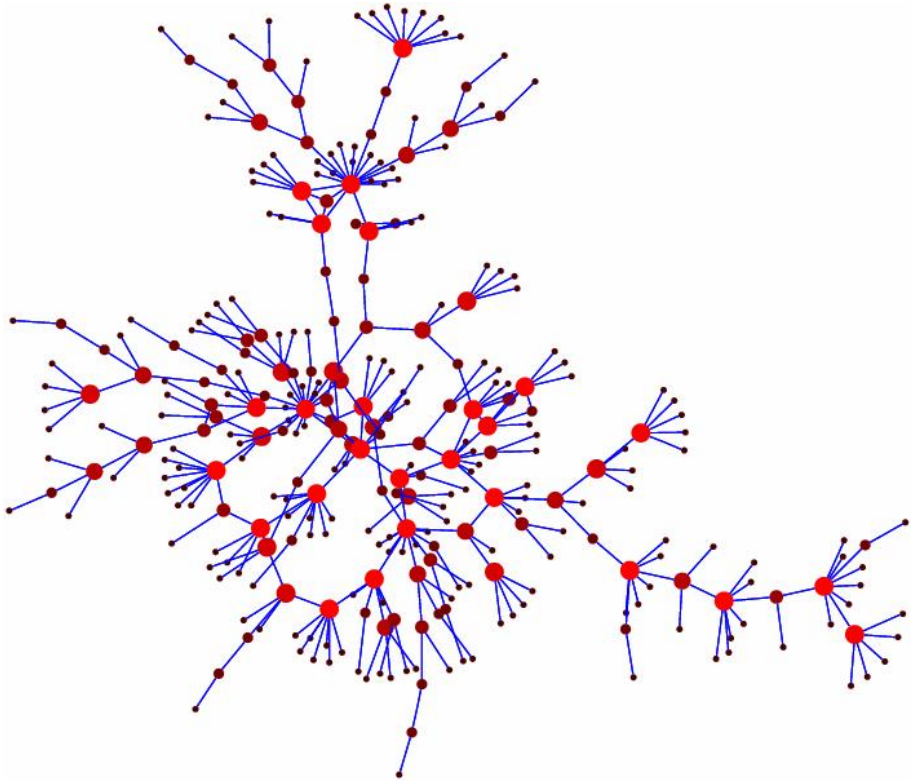
Complex systems

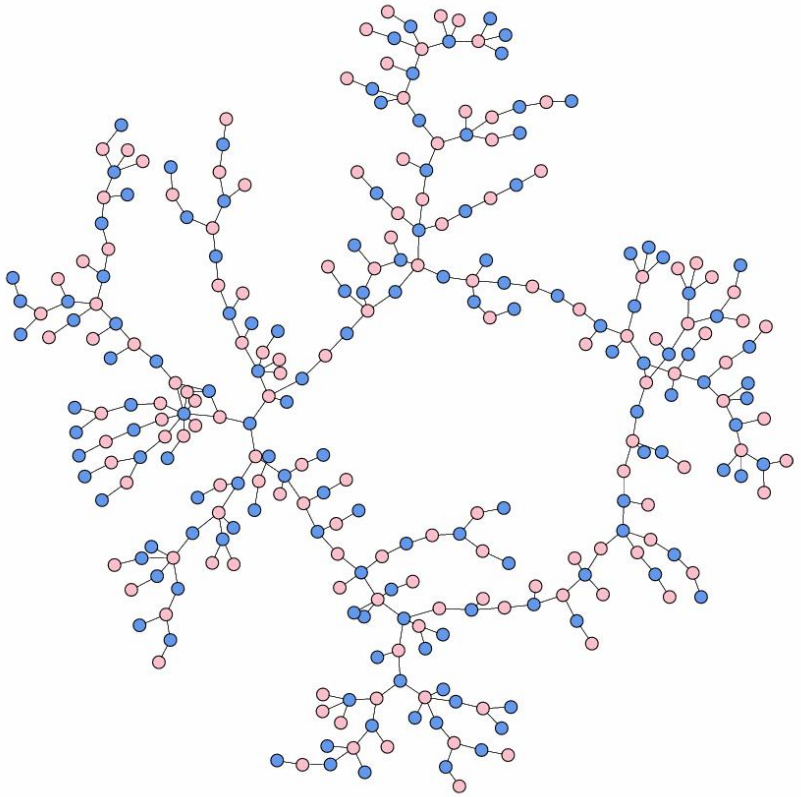
- Multiple length and time scales
- Irregular connectivity /
  Networked patterns of interaction
- Emergent behaviors
  - Self organization
    (e.g., patterns, synchronization)
  - Phase transitions
- Composite socio-technical systems
  - Power grids
  - Financial systems, etc.





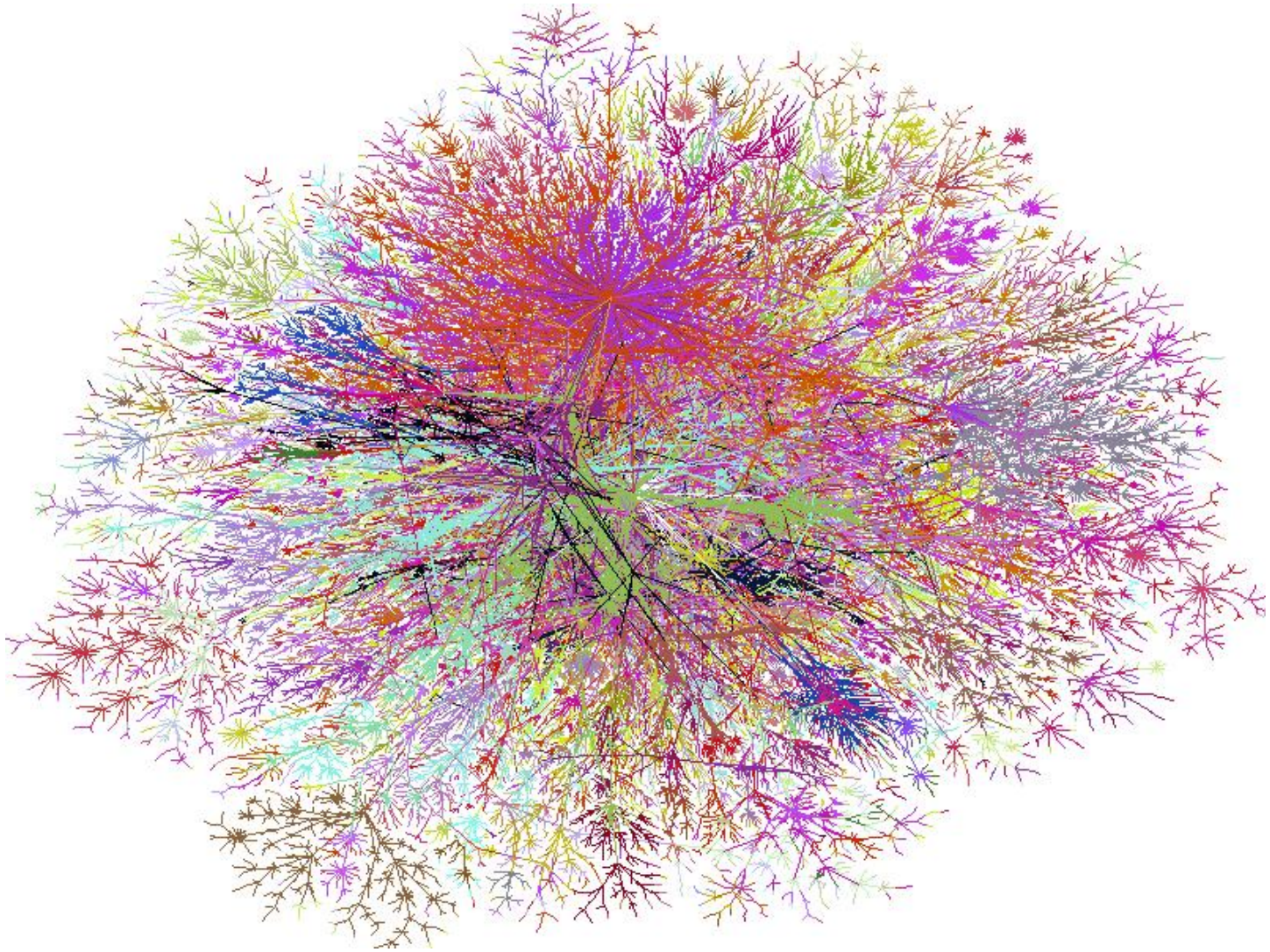
Example social networks
(Immunology; viral marketing; alliances/policy; economics)





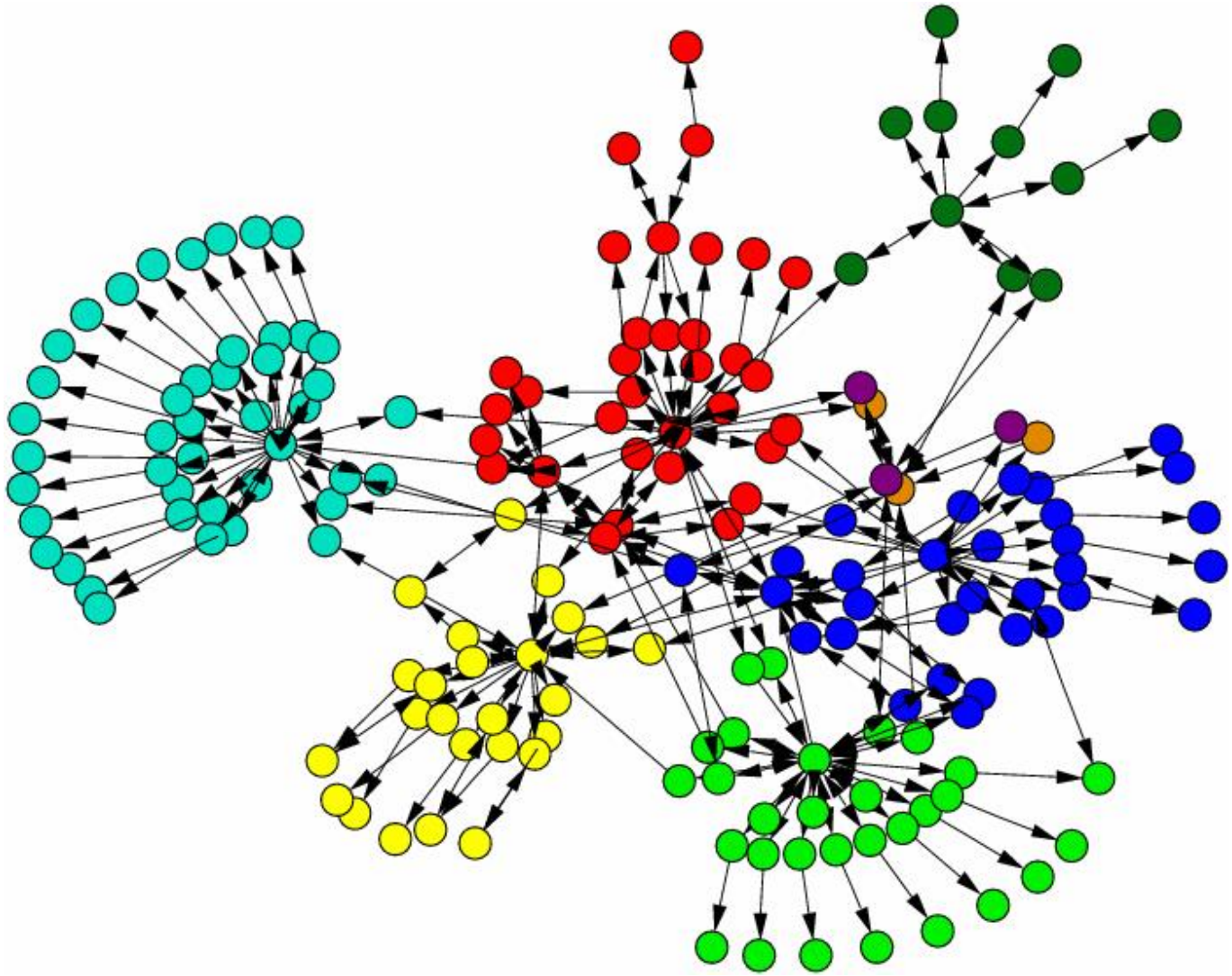
M. E. J. Newman

The Internet
(Robustness to failure; optimizing future growth; testing protocols on sample topologies)



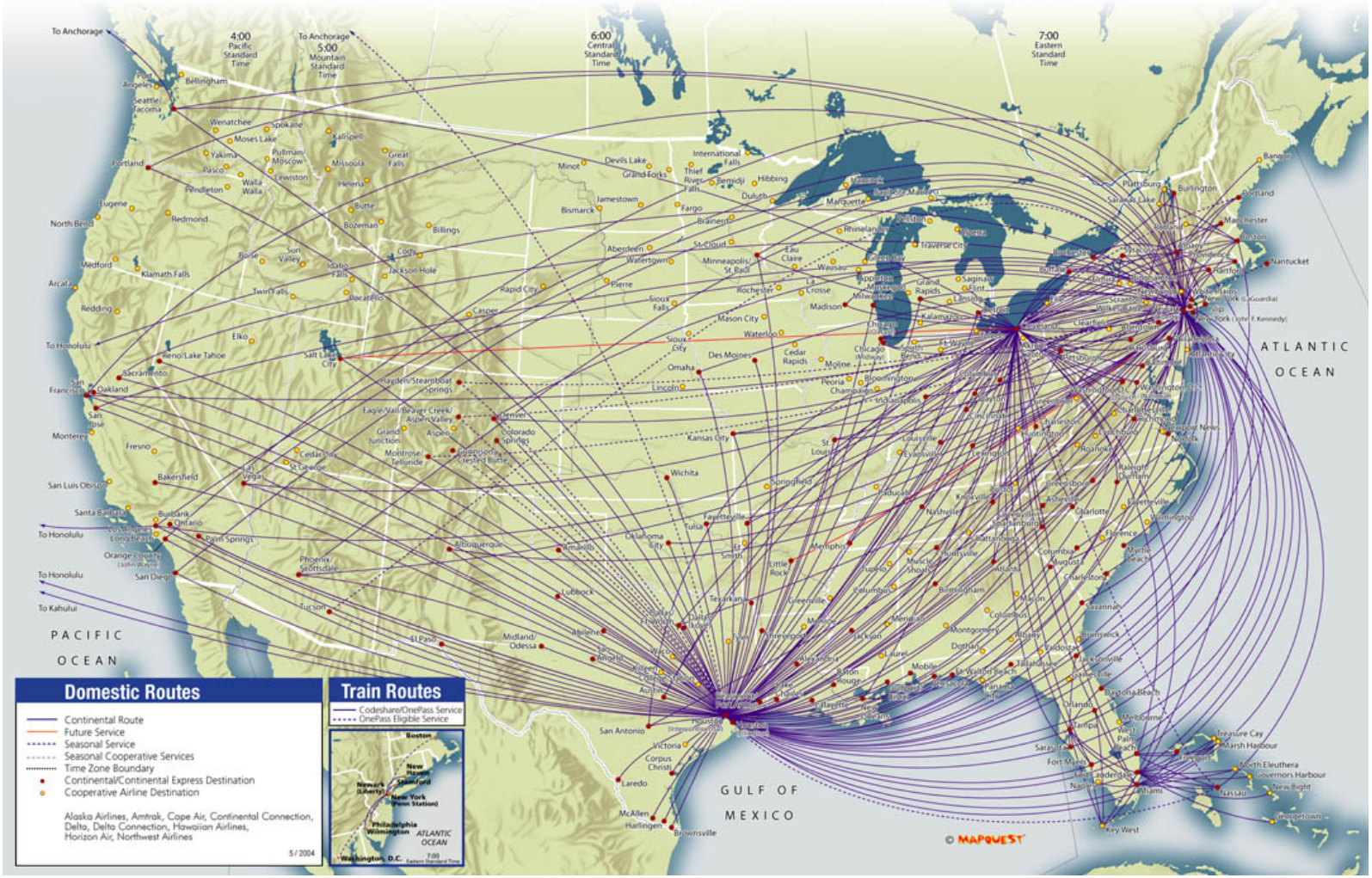
H. Burch and B. Cheswick

A typical web domain
(Web search/organization and growth)



M. E. J. Newman

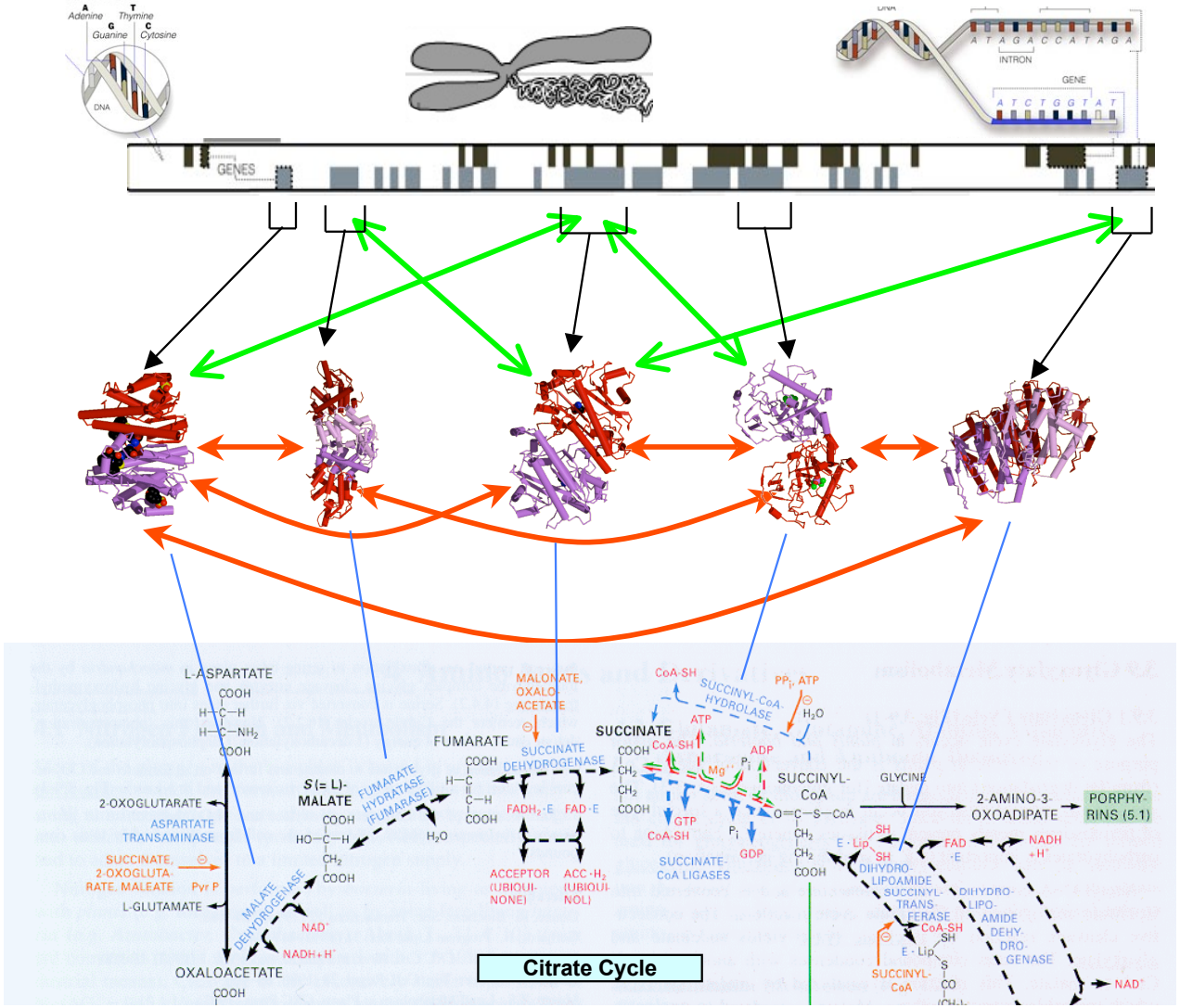
The airline network
(Optimization; dynamic demands and operating conditions)



Continental Airlines

Biology: Networks at many levels

Control mechanisms / drug design/ gene therapy / biomarkers of disease



GENOME

protein-gene interactions

PROTEOME

protein-protein interactions

METABOLISM

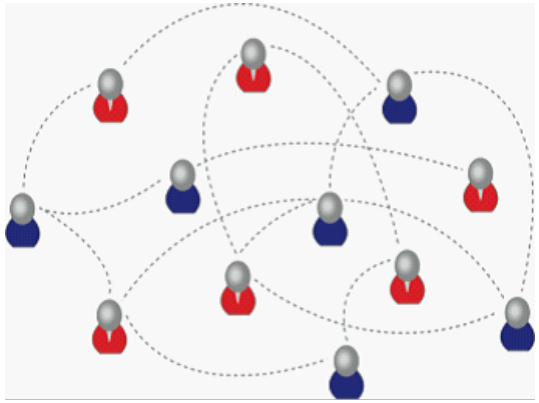
Bio-chemical reactions

Networks: Physical, Biological, Social

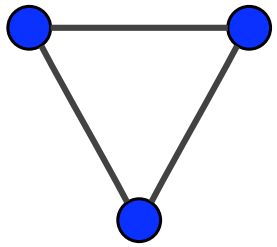
- **Geometric** versus **virtual** (Internet versus WWW).
- **Natural** /spontaneously arising versus **engineered** /built.
- **Directed** versus **undirected** edges.
- Each network **optimizes** something unique.
- Identifying **similarities** and fundamental **differences** can guide future design/understanding.
- Interplay of **topology** and **function** ?

Unifying features

1. **Small Worlds** (Diameter $\sim \log(N)$)

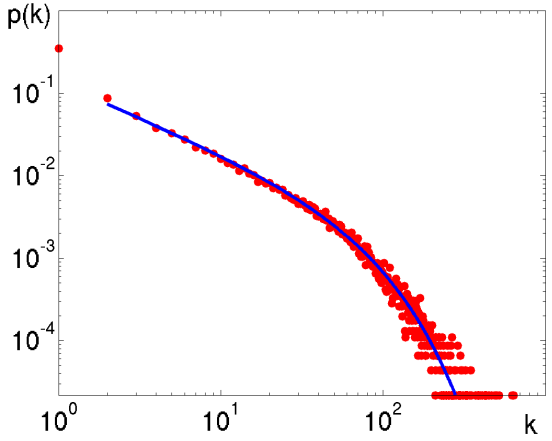


2. **Clustering in social networks** (Triangle motifs / transitive closure)



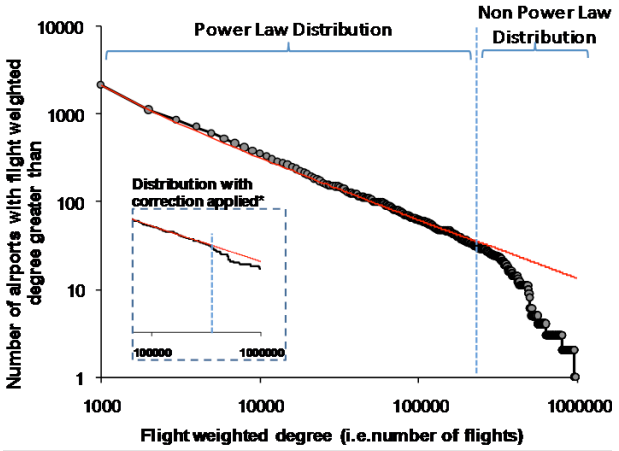
3. **Broad heterogeneity in node degree.**

Broad heterogeneity in node degree:
Approximate power-law degree distribution



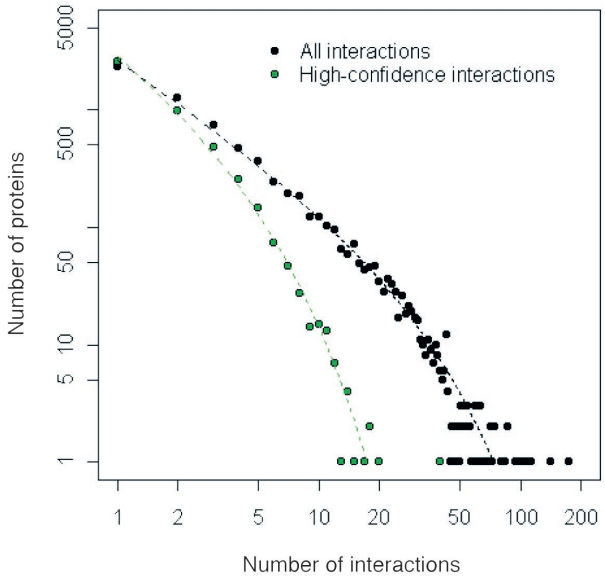
Social contacts

Szendröi and Csányi



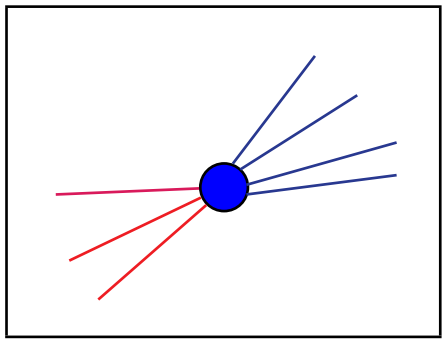
Airport traffic

Bounova 2009



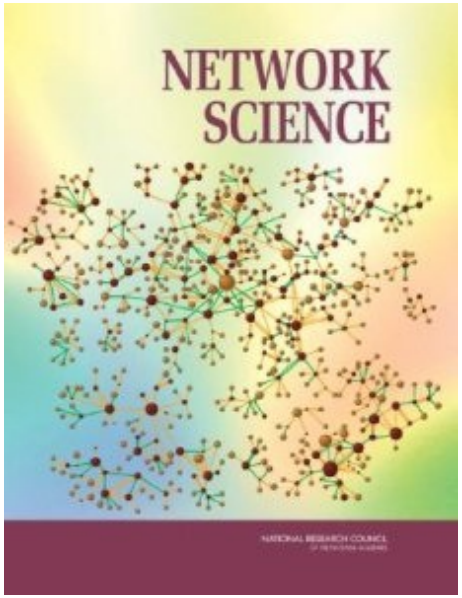
Protein interactions

Giot et al Science 2003



(node degree)

Natn Acam Sciences/Natn Research Council Study (2005)



"all our modern critical infrastructure relies on networks... too much emphasis on specific applications/jargon/disciplinary stovepipes... need a cross-cutting science of networks... Research for the 21st century"

Today's most pressing global problems

Global Agenda Council on Complex Systems 2012-2014

WORLD ECONOMIC FORUM

COMMITTED TO IMPROVING THE STATE OF THE WORLD

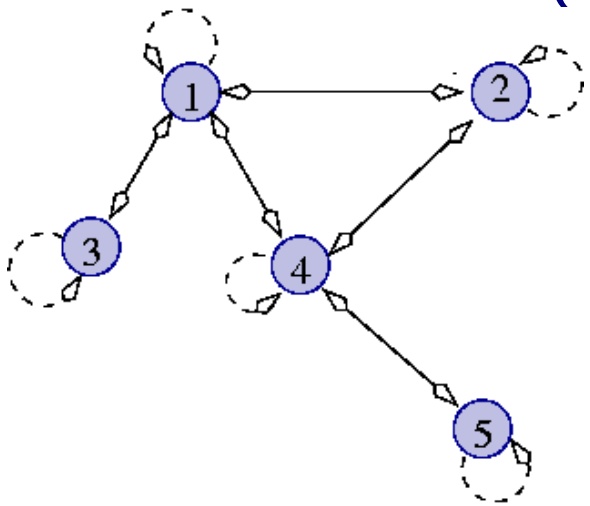
"Our hyper-connected, interdependent world"

How do we represent a simple individual network as a mathematical object?

Representing a network: NETWORK TOPOLOGY

Connectivity matrix, M:

$$M_{ij} = \begin{cases} 1 \text{ if edge exists between } i \text{ and } j \\ 0 \text{ otherwise.} \end{cases}$$



$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} = M$$

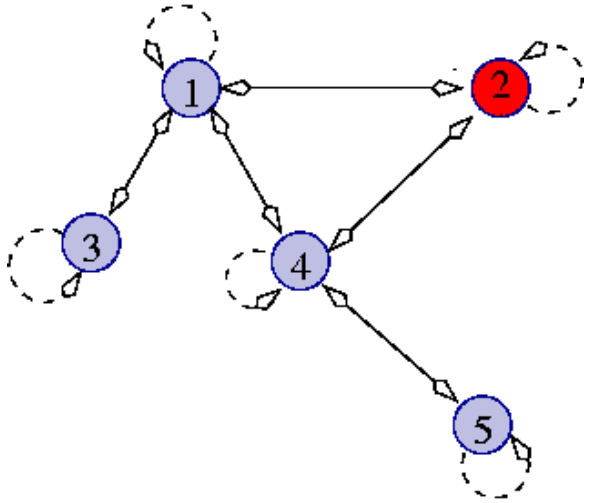
Node **degree** is number of links.

(Diameter, Clustering, Assortativity, Betweenness centrality)

Network Activity: FLOWS on NETWORKS

(Spread of disease, routing data, materials transport/flow)

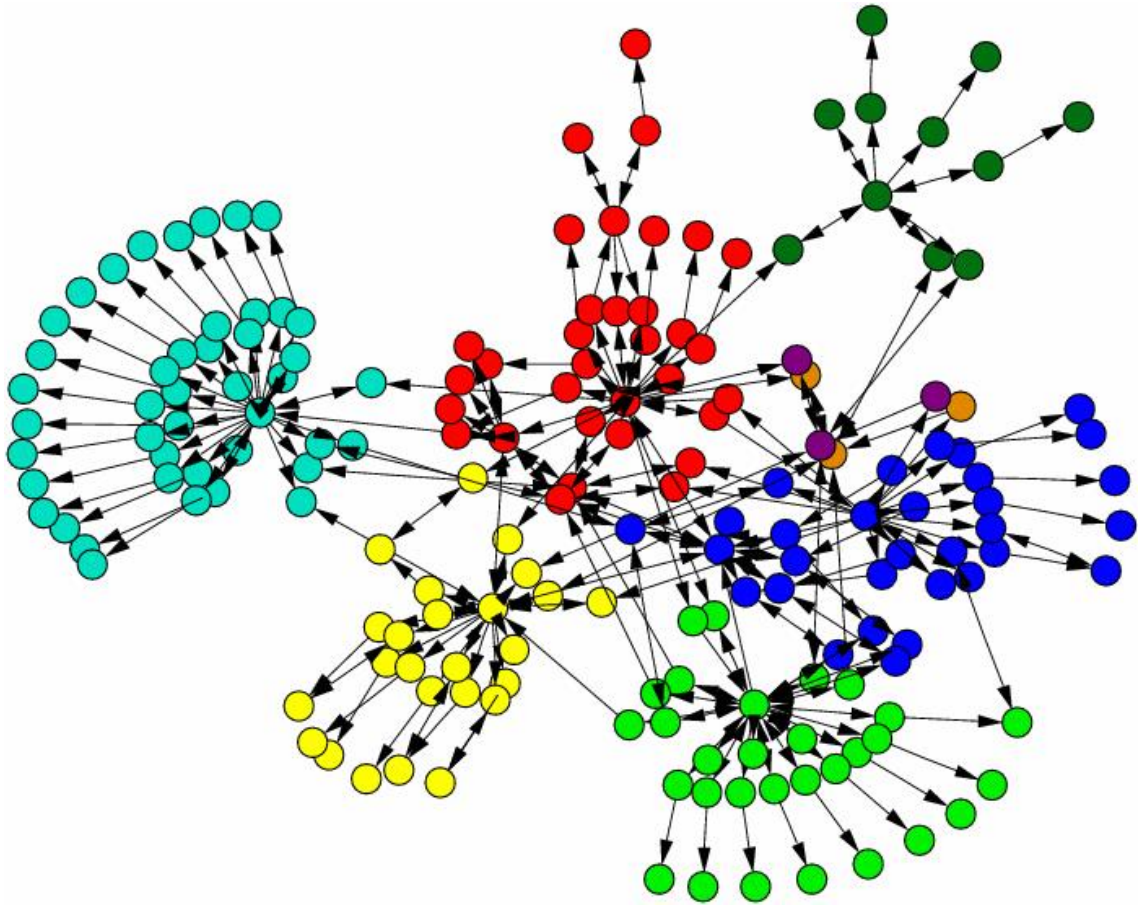
Random walk on the network has state transition matrix, P:



$$\begin{pmatrix} 1/4 & 1/3 & 1/2 & 1/4 & 0 \\ 1/4 & 1/3 & 0 & 1/4 & 0 \\ 1/4 & 0 & 1/2 & 0 & 0 \\ 1/4 & 1/3 & 0 & 1/4 & 1/2 \\ 0 & 0 & 0 & 1/4 & 1/2 \end{pmatrix} = P$$

- Eigenvectors $\rightarrow$ **Page Rank**, Communities/modules.
- Eigenvalues $\rightarrow$ timescales.

Random walk on the WWW is the "Page Rank" underlying Google's search technology

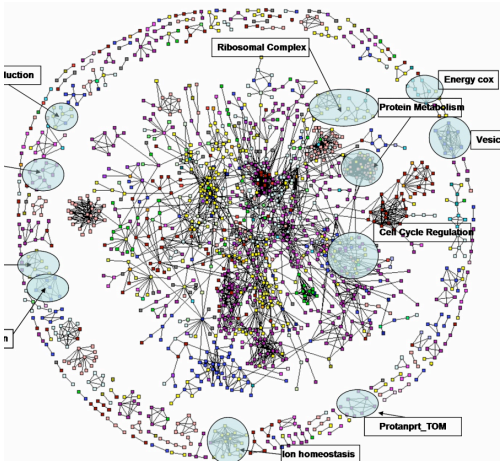


Page Rank of a node is the steady-state random walk occupancy probabilty.

Modeling the ubiquitous networks of modern society?

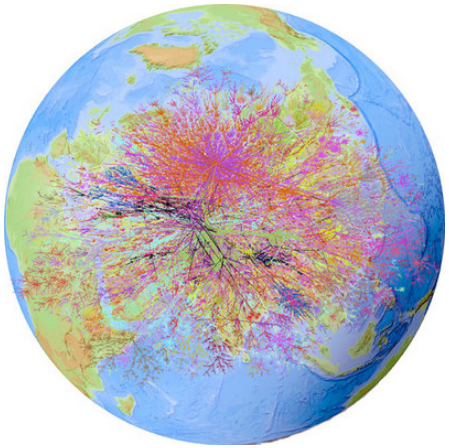
Networks:

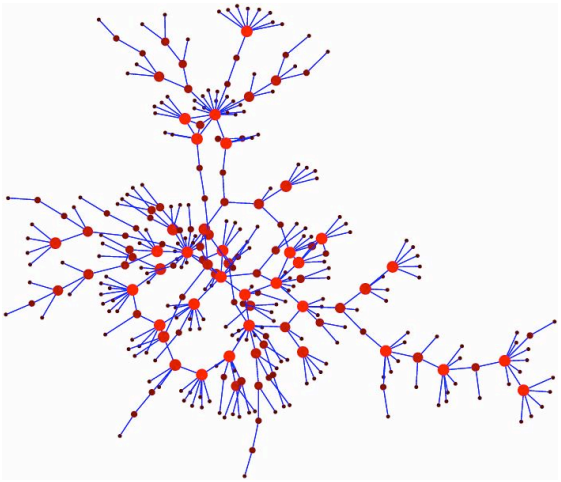
Transportation Networks/ Power grid
(distribution/ collection networks)



Biological networks
- protein interaction
- genetic regulation
- drug design

Computer networks



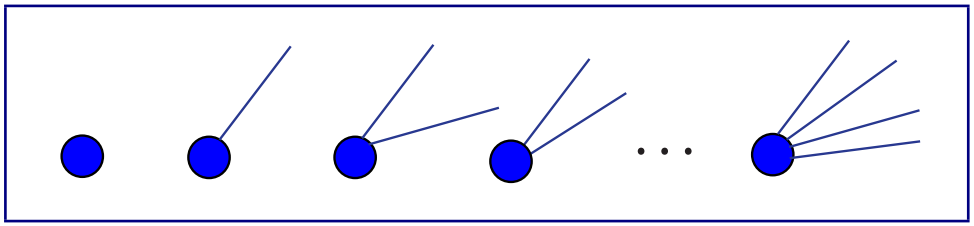


Social networks
- Immunology
- Information
- Commerce

Modeling networks as random graphs

Statistical Physics Approach

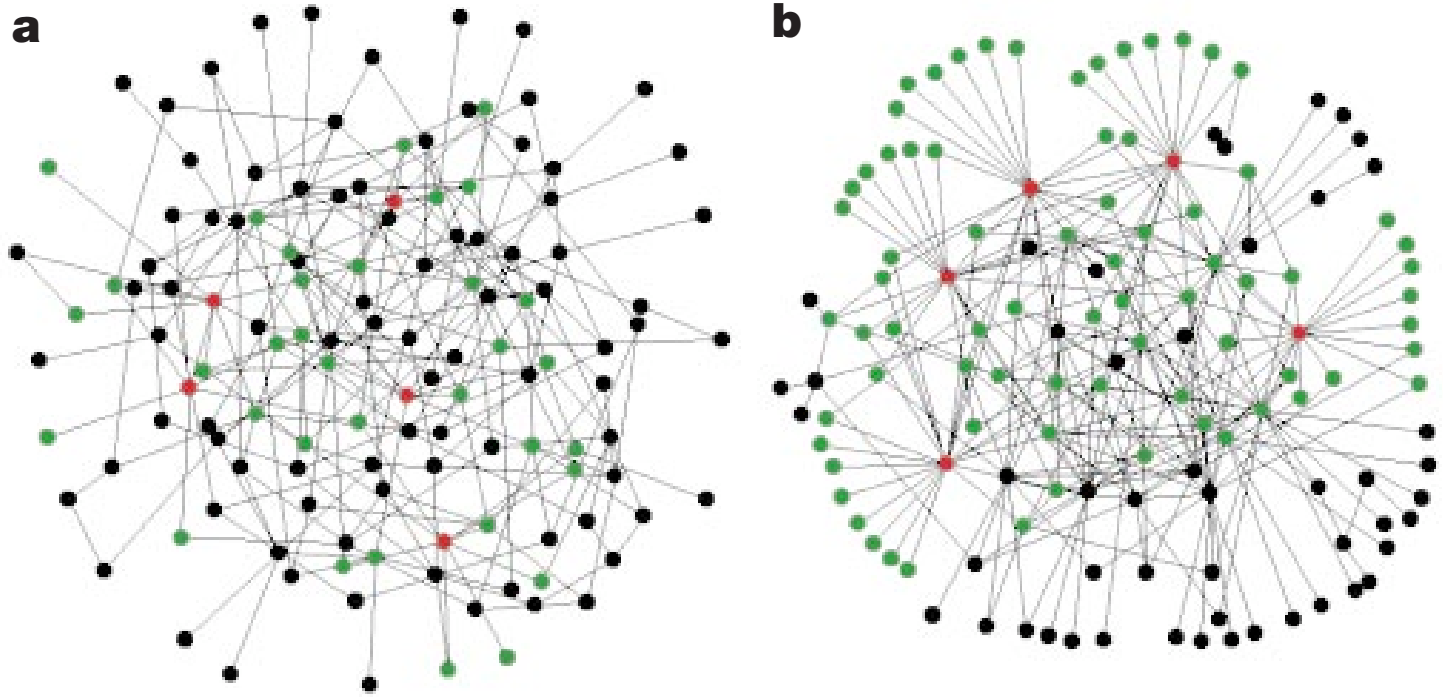
- **Erdős and Rényi** random graphs (1959, 1960). Phase transition.
- **Configuration models** (Bollobás 1980, Molloy and Reed *RSA* 1995). Enumerating over all networks with specified $\{p_i\}$.



- **Preferential attachment** (Barbási-Albert 1999, etc.)
- **Growth by copying** (Kumar, Raghavan, Rajagopalan, Sivakumar, Tomkins, Upfal *FOCS* 2000), including duplication/mutation (Vazquez, Flammini, Maritan, Vespignani, *ComPlexUs* 2003)

Universal properties of the **ensemble** of graphs.

Random graphs with power law degree distributions



Exponential

Scale-free

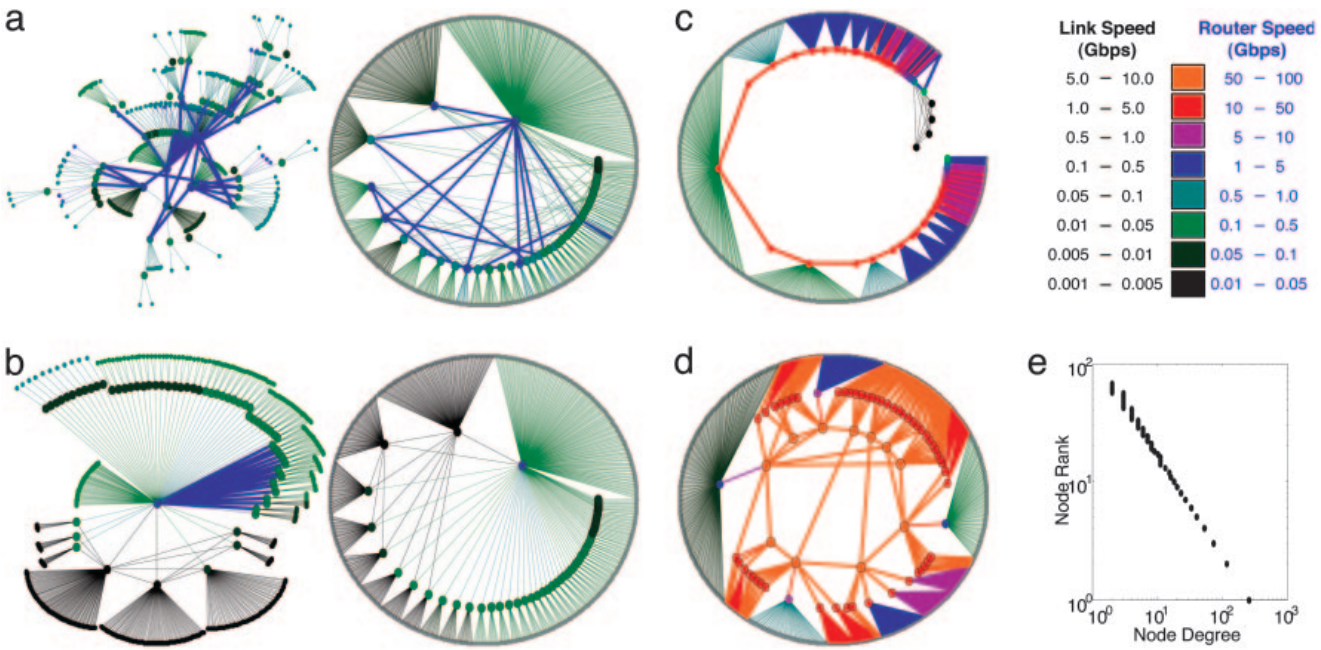
- **Fragile:** Quickly loose connectivity if remove “hubs” (red nodes).
- **Robust:** Stay connected if remove large fraction of randomly chosen nodes

“Robust yet Fragile”

Random graphs versus an individual engineered or biological system

- **Ensemble not necessarily representative!**
  Doyle, et. al., PNAS **102** (4)2005.

All these have same deg dist, p_i:

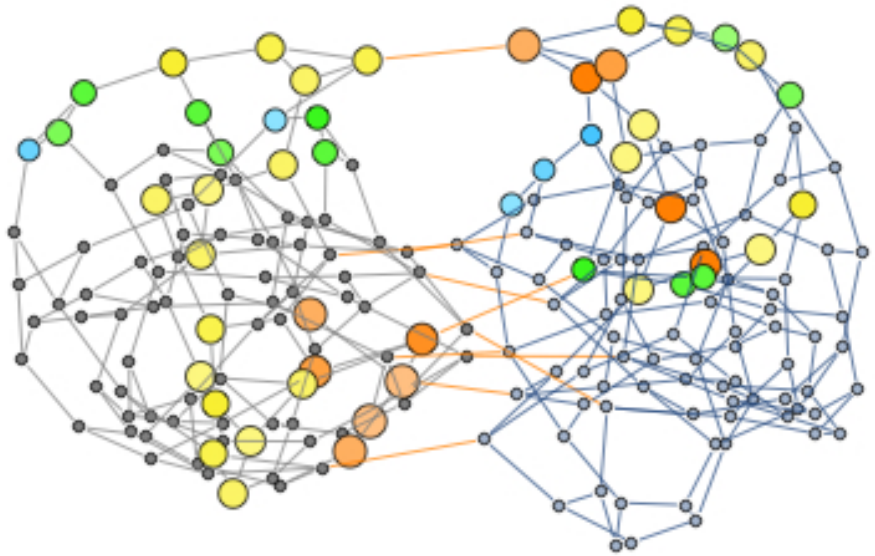


- Neglects design principles: Redundancy, degree correlations, local optimization (Although D'Souza, et. al. PNAS 2007), ...

Today's talk: Random graph models of networks
Two technical results

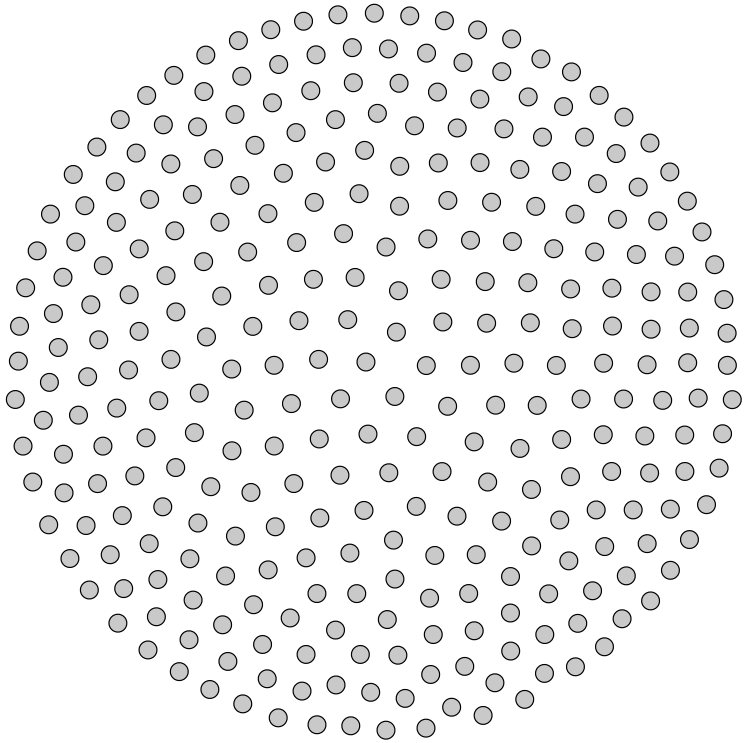
- Part I. Dynamics **of** networks
  - Explosive percolation phase transitions

- Part II. Dynamics **on** interdependent networks
  - Optimal interdependence



The "classic" random graph, $G(n, p)$

- P. Erdős and A. Rényi, "On random graphs", *Publ. Math. Debrecen.* 1959.
- P. Erdős and A. Rényi, "On the evolution of random graphs", *Publ. Math. Inst. Hungar. Acad. Sci.* 1960.
- E. N. Gilbert, "Random graphs", *Annals of Mathematical Statistics*, 1959.

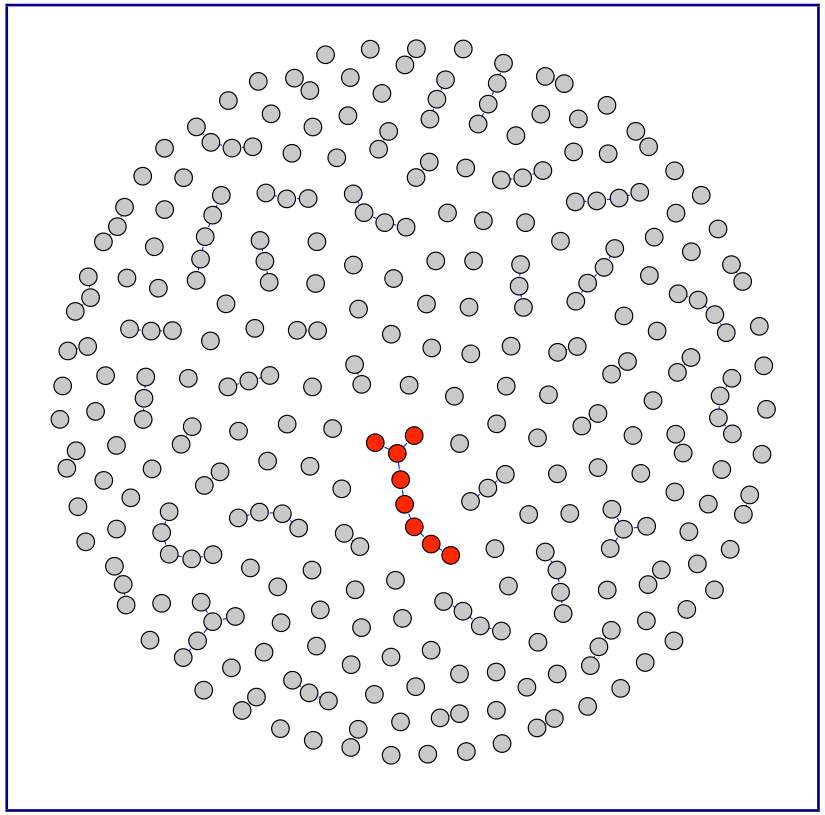


- Start with n isolated vertices.
- Consider each possible edge, and add it with probability p.

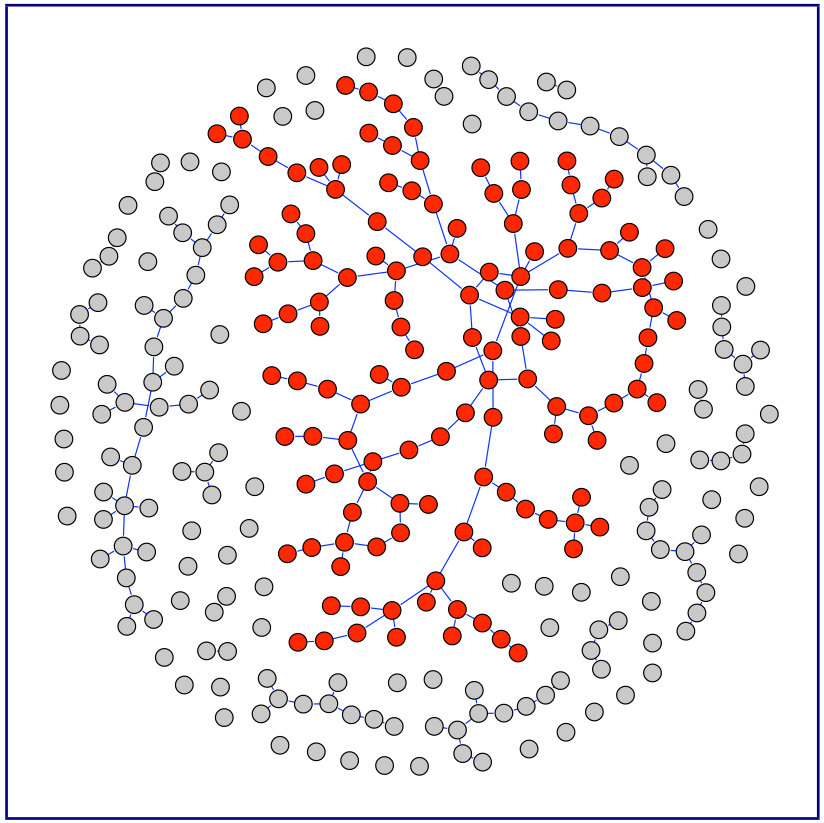
What does the resulting graph look like?

(Typical member of the ensemble)

G(n=300,p)

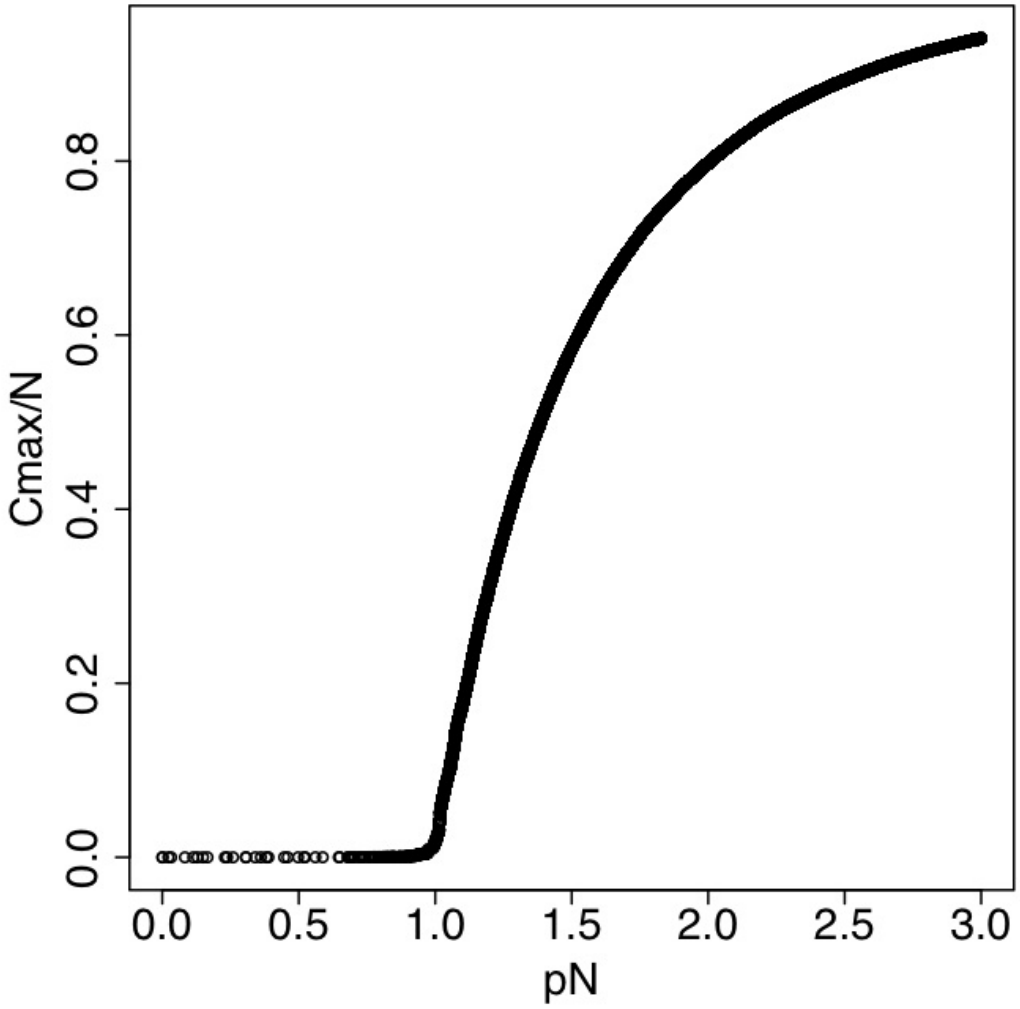


$p = 1/400 = 0.0025$



$p = 1/200 = 0.005$

Emergence of a *unique* "giant component"
Phase transition in connectivity



- $p_c = 1/n.$
- $p < p_c$, $C_{\max} \sim \log(n)$
- $p = p_c$, $C_{\max} \sim n^{2/3}$
- $p > p_c$, $C_{\max} \sim A \cdot n$

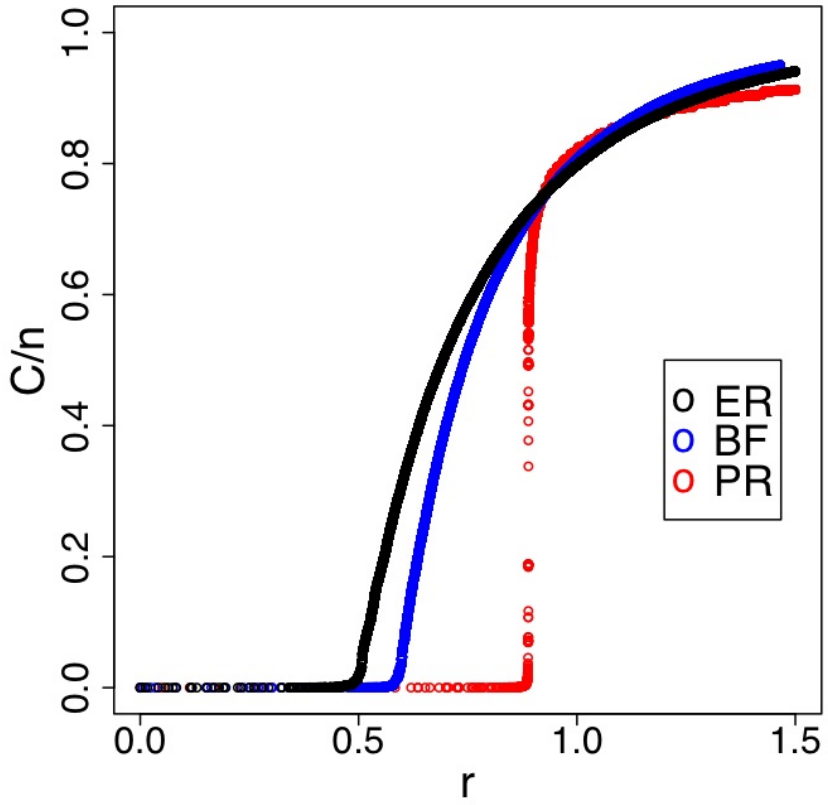
Expected # of edges per node

$$t = e/n = p(n-1)/2$$

so $\boxed{t_c = 1/2}$

Is connectivity a good thing?

Is connectivity a good thing?
(Context dependence)



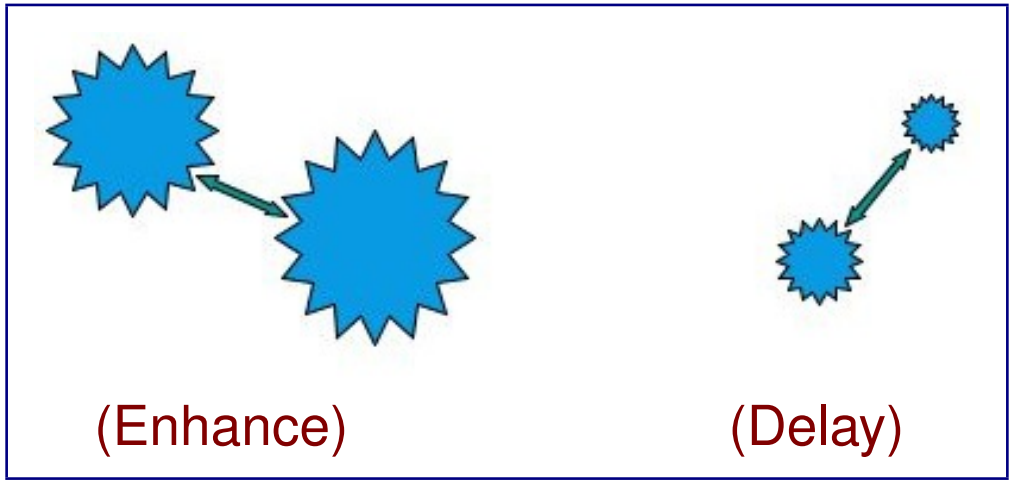
- Communications, Transportation, Synchronization, ... versus
- Spread of human or computer viruses

Can any limited perturbation change the phase transition?

[Bohman, Frieze, *RSA* **19**, 2001]

[Achlioptas, D'Souza, Spencer, *Science* **323**, 2009]

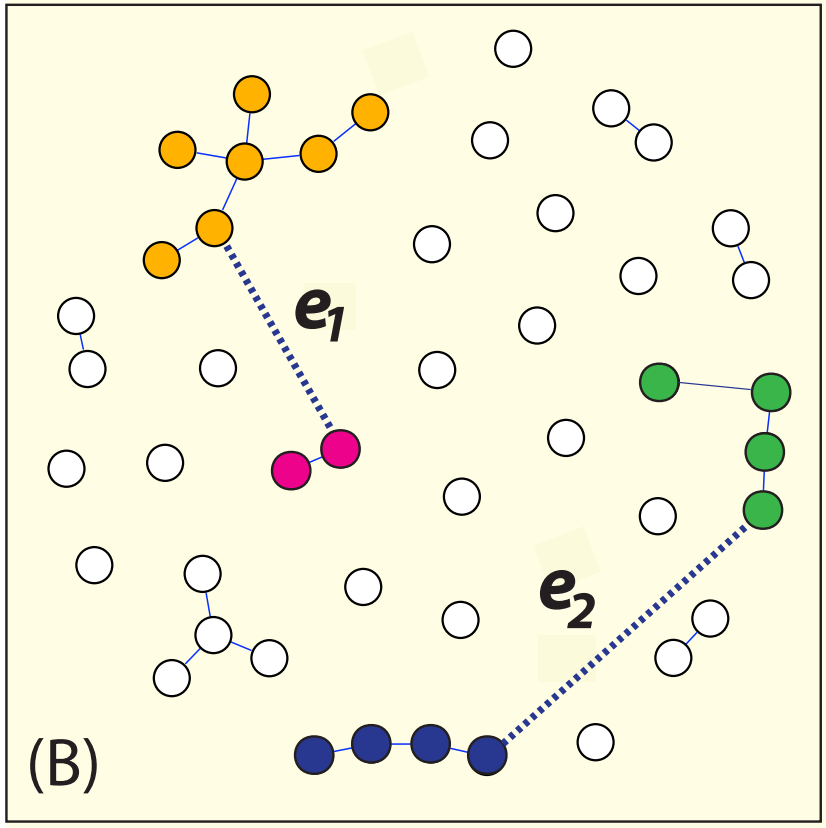
- Possible to **Enhance** or **Delay** the onset?
- The "**Product Rule**"
  - Choose ***two*** edges at random each step.
  - Add only the desirable edge and discard the other.



- The Power of Two Choices in randomized algorithms.

  Azar; Broder; Mitzenmacher; Upfal; Karlin;

ProdRule: Explicit example

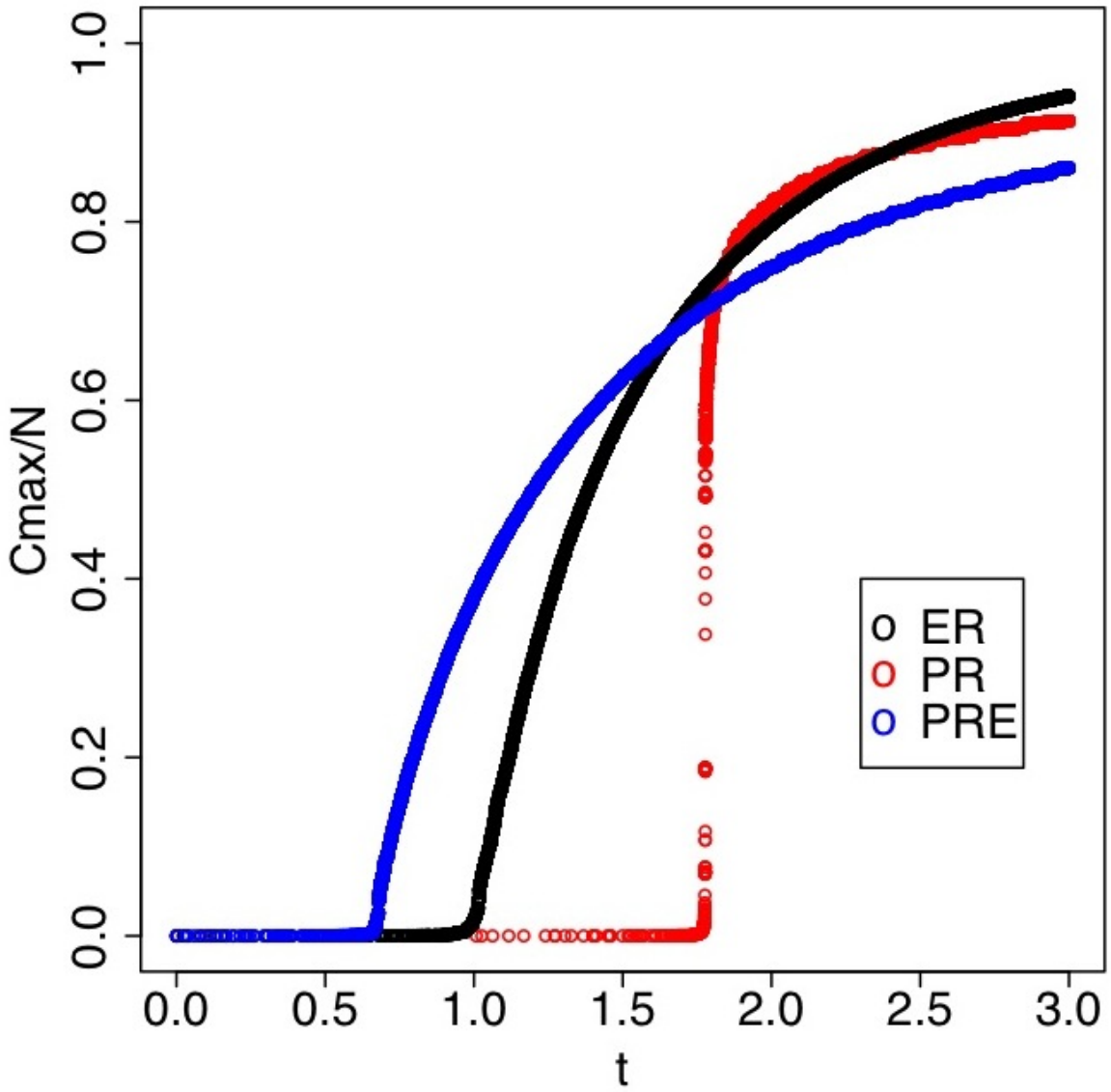


- Prod $e_1 = (7) \times (2) = 14$
- Prod $e_2 = (4) \times (4) = 16$
- To *enhance* choose e_2. To *delay* choose e_1.

Product Rule

- **Enhance** – similar to **ER** but with earlier onset.

- **Delay** – Extremely abrupt



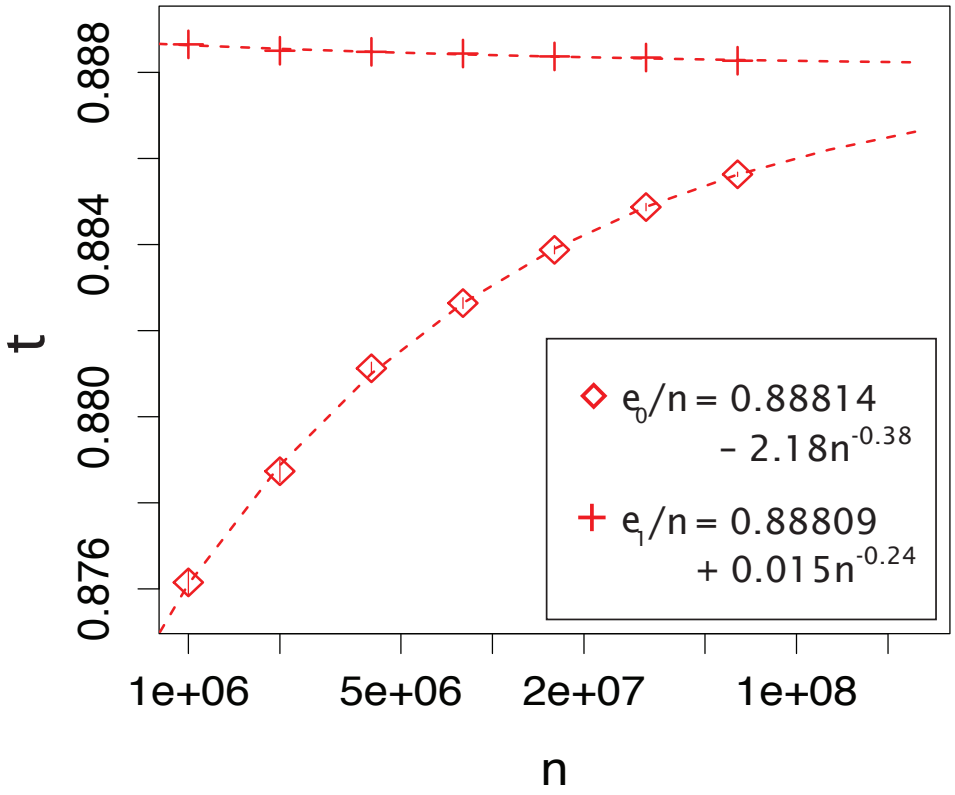
In terms of edge density or "time", t_c, where $t = e/n$

(Note, for ER, $t_c = 1/2$)

- For $t < t_c$, $C_{\max} < n^{1/2}$.
- For $t > t_c$, $C_{\max} > 0.5n$.

Jumps "instantaneously"

from $C_{\max} = n^{1/2}$ to $0.5n$.



"Explosive Percolation in Random Networks"

Achlioptas, D'Souza, Spencer, *Science*, **323** (5920), 2009

Why this is surprising

Percolation theory, a theoretical underpinning for :

- Onset of epidemic spreading
- Flow through porous media / random transport
- Vulnerability and resilience of networks

– Many prior variants (bond, site, directed, ...) on many types of networks and lattices; All continuous transitions.

– **Continuous:** accompanied by *critical scaling* which can provide warning signs.

– **Abrupt:** detection, intervention, recovery more challenging, esp. cascading failures.

Many more EP systems and mechanisms now discovered

(Condensed list here)

Lattice percolation, power law graphs, cluster aggregation:

- R. Ziff, *Phys. Rev. Lett.* 103, 045701 (2009).
- Y. S. Cho, J. S. Kim, J. Park, B. Kahng, D. Kim, *Phys. Rev. Lett.* 103, 135702 (2009).
- F. Radicchi, S. Fortunato, *Phys. Rev. Lett.* 103, 168701 (2009).
- E. J. Friedman, A. S. Landsberg, *Phys. Rev. Lett.* 103, 255701 (2009).
- Y.S. Cho, B. Kahng, D. Kim, *Phys. Rev. E* (R), 2010.
- R. M. D'Souza, M. Mitzenmacher, *Phys. Rev. Lett.* 104, 195702 (2010).
- Araújo, Andrade Jr, Ziff, Herrmann, *Phys. Rev. Lett.* 106, 095703 (2011).
- Hooyberghs, Van Schaeybroeck, Phys. Rev. E 83, 032101 (2011).
- Gomez-Gardenes, Gomez, Arenas, Moreno, *Phys. Rev. Lett.* 106, 128701 (2011).

Observed in real world:

- Rozenfeld, Gallos, Makse; *Eur. Phys. J. B*, 75, 305-310, (2010). (PHN)
- Pan, Kivelä, Jari Saramäki, Kaski, Kertész, Phys. Rev. E 83, (2011). (Communities)
- Y. Kim, Y.-k. Yun, and S.-H. Yook, Phys. Rev. E 82, 061105 (2010). (Nanotubes)
- Y. S. Cho and B. Kahng. Phys. Rev. E, 84, 050102 (2011). (DLA)
- Growth of Wikipedias (Bounova, personal communication.)

Alternate mechanisms (with out edge competition):

- Araújo, Herrmann, *Phys. Rev. Lett.* 105, 035701 (2010).
- W. Chen, R. M. D'Souza, *Phys. Rev. Lett.* 106, 115701 (2011).

Mechanisms underlying truly discontinuous percolation

(Suppressing the growth of large clusters)

- **Fixed choice on a lattice of d $<$ 6** , Cho et al. *Science* 338, 2013.

- **Control only of the largest cluster**

  – Araujo, N. A. M. & Herrmann, H. J. , et al.
  Phys. Rev. Lett. 105, 035701 (2010), *Phys. Rev. Lett.* 106, (2011).

  – W. Chen & R.D. *Phys. Rev. Lett.* 83 (2011); Chen, Zheng, R. D. *EPL* 100 (2012).
  Growth by overtaking.
  Common in ecology and business.

- **Cooperative/correlated phenomena**

  – Bizhani, Paczuski, Grassberger,
  PRE 86 (2012) [-3mm]

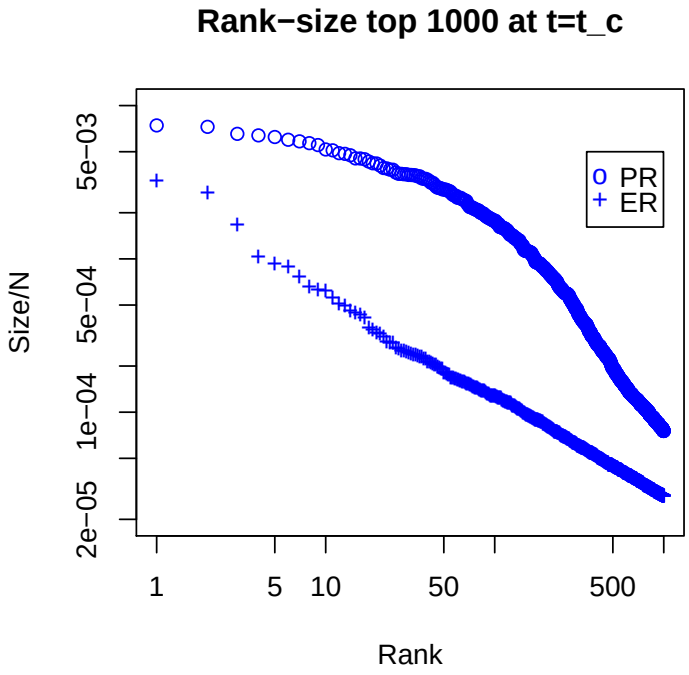
  – L. Cao, J. M. Schwarz, *PRE* 86 (2012).

- **Dressing up a simple structure**

  Boettcher, Singh, Ziff, *Nature Comm.*, 3:787 (2012).

- **Restricted Erdős-Rényi**

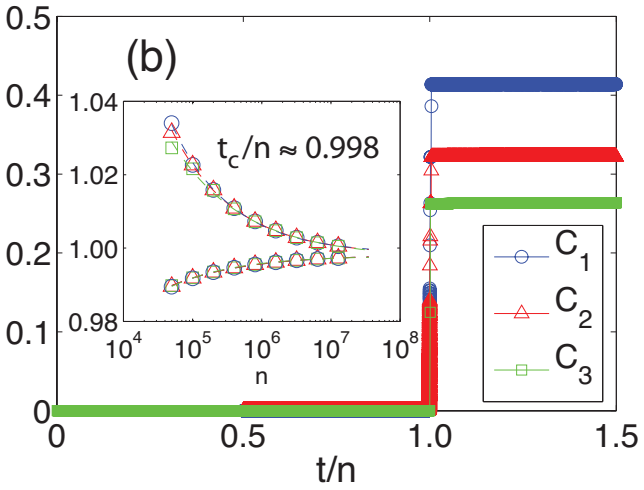
  Panagiotou, et. al. *Elec. Notes. Disc. Math.* 2011.



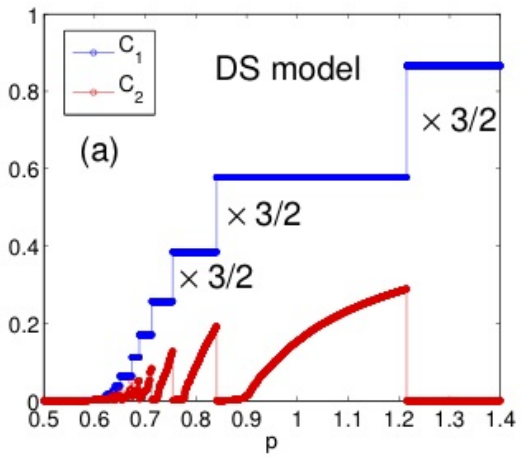
Additional esoteric behaviors of EP systems

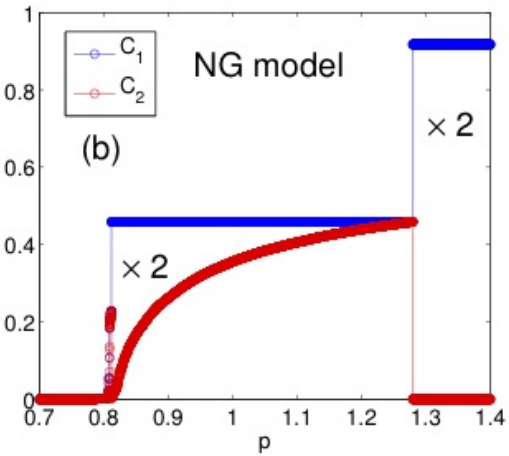
- **Multiple giant components**

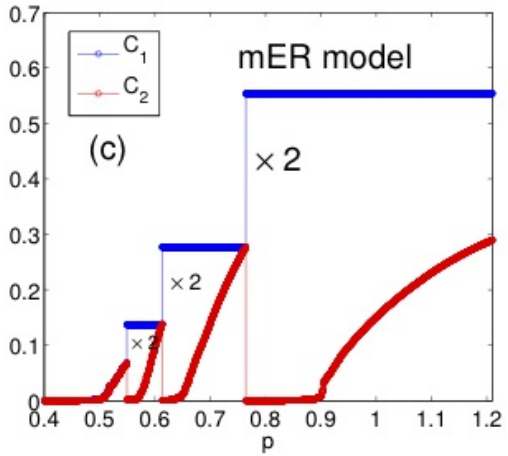
W. Chen, R. D., *Phys. Rev. Lett.* **106** 115701 (2011).



- **An (infinite) series of discontinuous transitions possible**





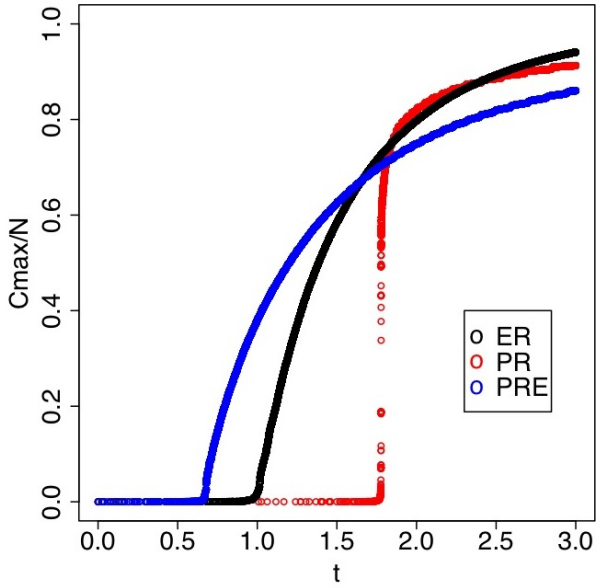


– Nagler, Tiessen, Gutch, *Phys. Rev. X* **2**, 031009 (2012).

– Chen, Cheng, Zheng, Chung, R.D., Nagler, *Phys Rev E* **88**, 042152 (2013).

Explosive Percolation

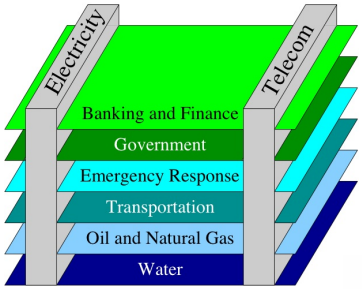
- Apply a patch repeatedly to delay onset of percolation
- Explosive onset when inevitable ultimately occurs
- Intervention impossible: Once detected, too late



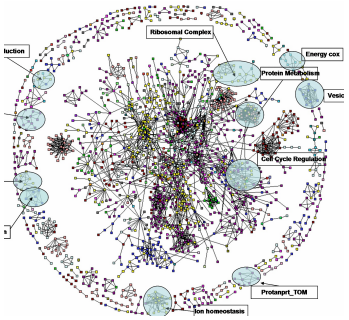
- A new paradigm for modeling extreme events in engineered systems, from dams to financial markets; e.g. Helbing *Nature*, 2013
- "Coupled Networks, Dragon Kings and Explosive Percolation: New Views on Extreme Events", Workshop of the ETH Risk Center, ETH Zurich, October 25-26th, 2012.

From single to interacting networks

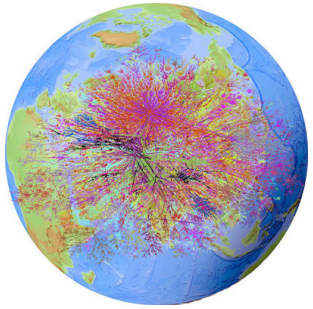
(Recall, even a single network has emergent properties)



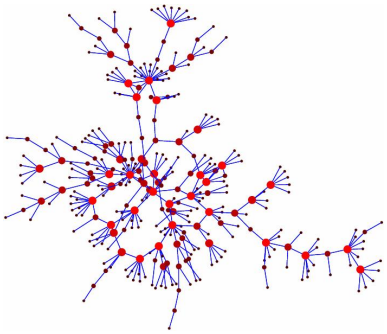
Critical Infrastructure



Biological & Ecological networks



Information and Communication technology

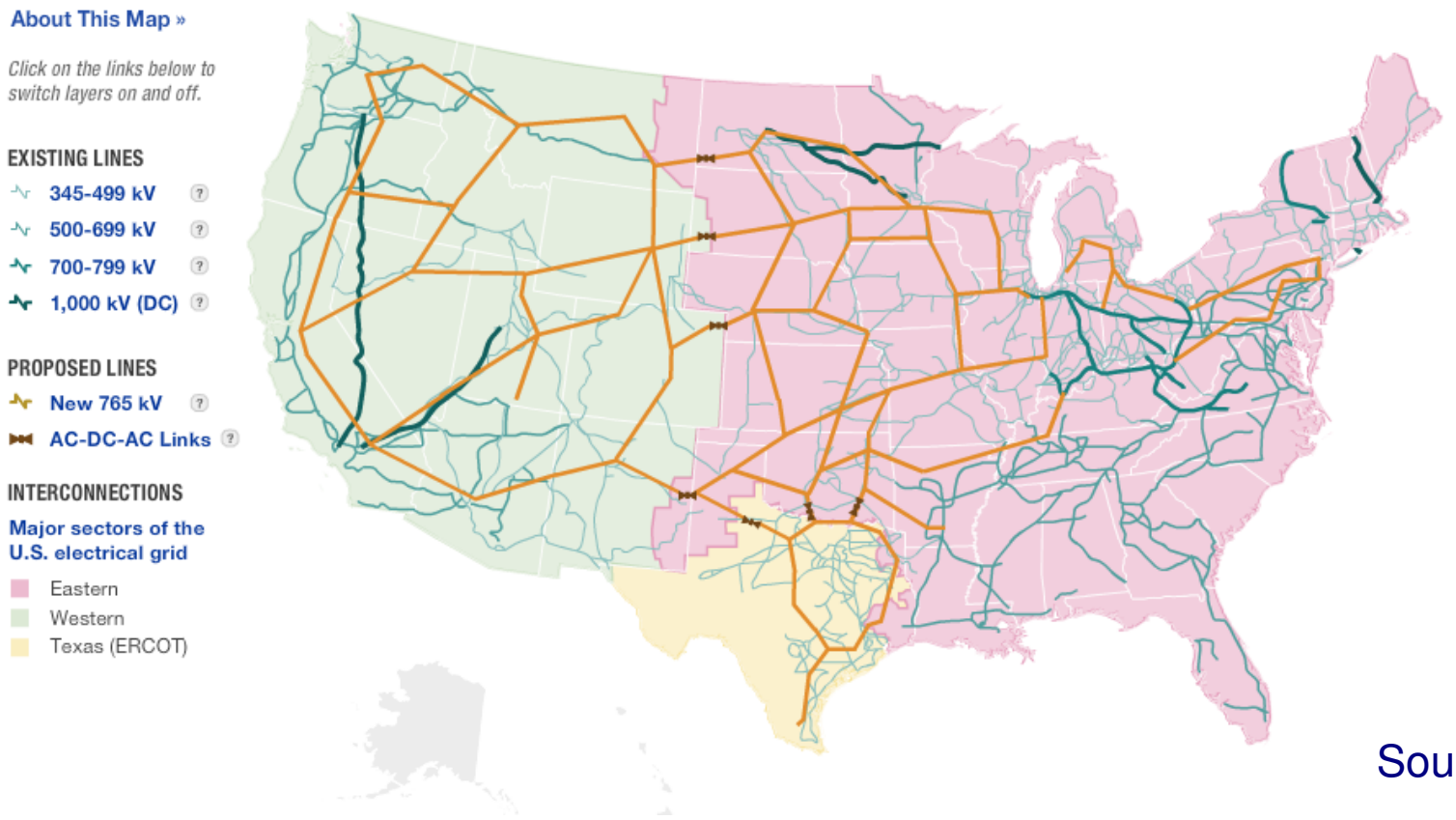


Social networks: Economics & Epidemics

- E-commerce (incl social networks/marketing) $\rightarrow$ WWW $\rightarrow$ Internet $\rightarrow$ Supply chains $\rightarrow$ Power grid $\rightarrow$ River networks.
- Biological virus $\rightarrow$ Social contact network $\rightarrow$ Transportation networks $\rightarrow$ Communication networks $\rightarrow$ Power grid $\rightarrow$ River networks.

Part II: Dynamics on interdependent networks

The “power grid” is a collection of independently owned and operated plants and transmission lines.

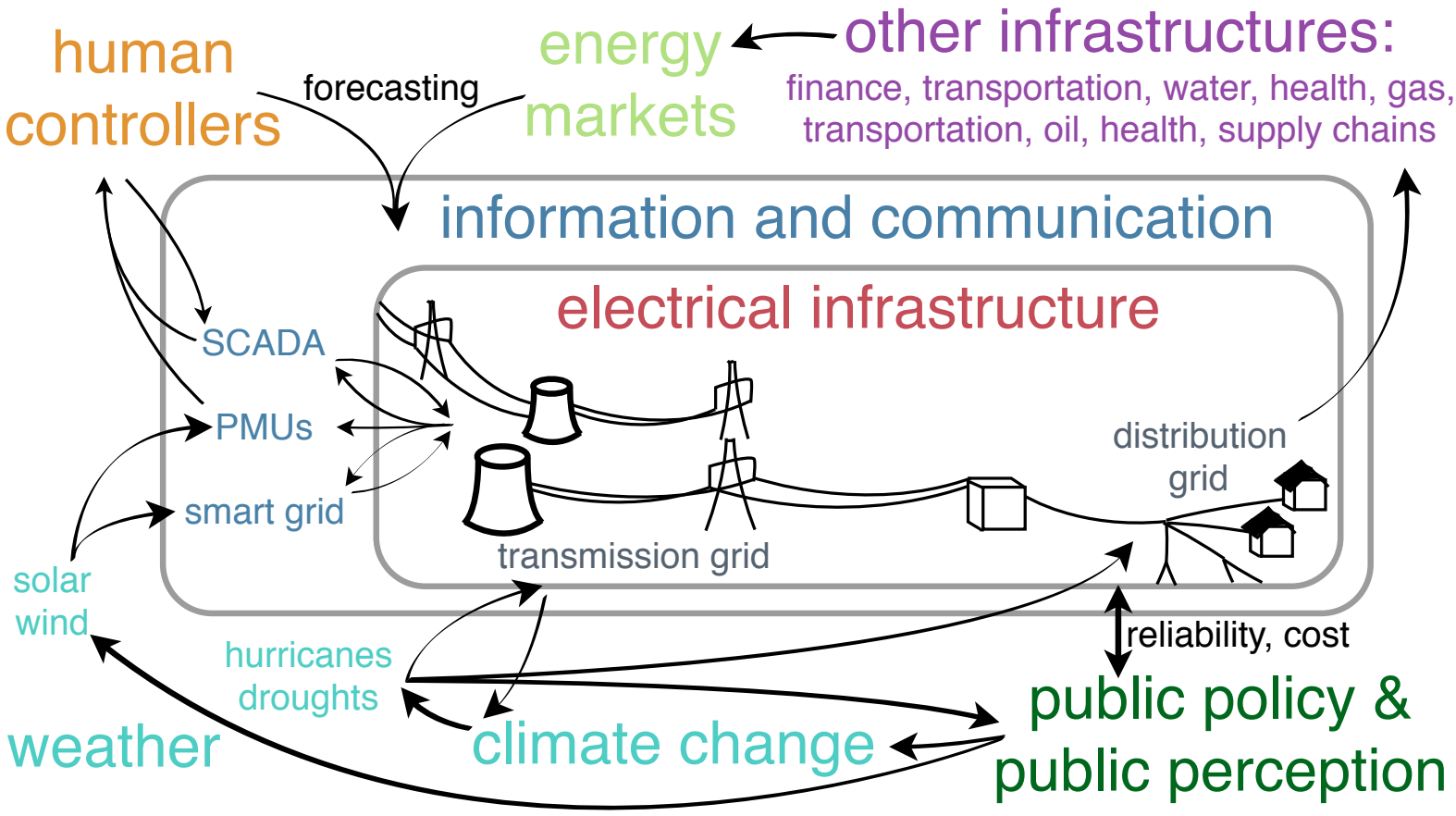


Source: NPR

- We can’t store power. Need supply=demand.
- Blackouts cascade from one grid to another.
- Increasingly distributed and interconnected (in both US and Europe)

Caveat: Real power systems – a web of feedbacks

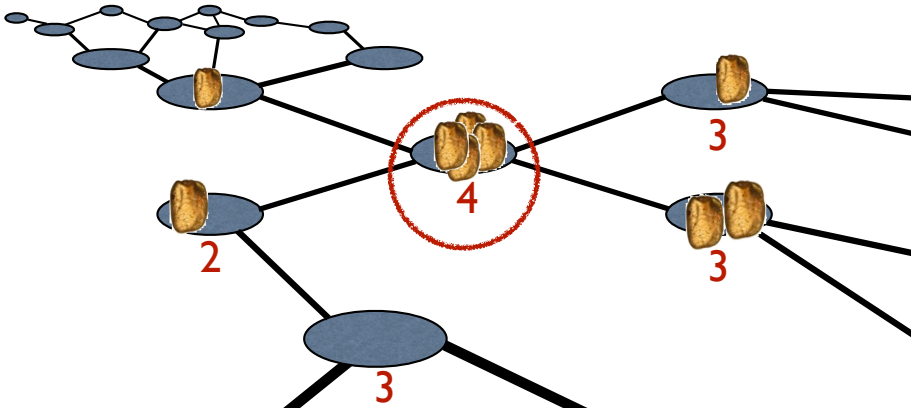
Brummitt, Hines, Dobson, Moore, R.D., *PNAS* July 23, 2013.
"Transdisciplinary electric power grid science"



Bak-Tang-Wiesenfeld *PRL* 1987: self-organized criticality

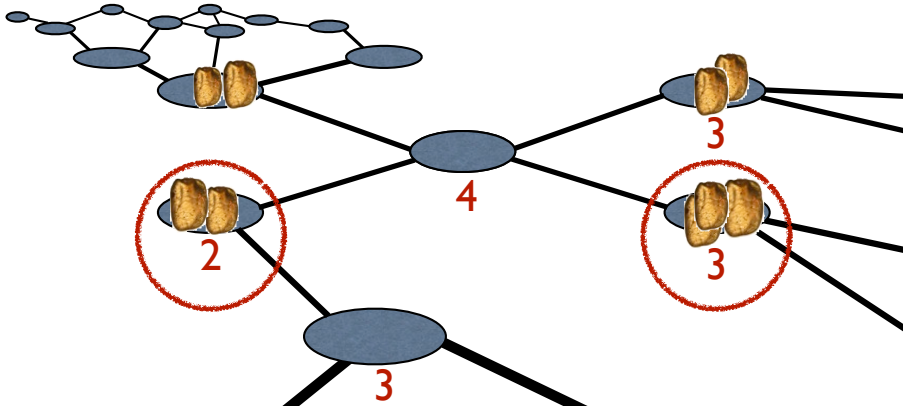
Sandpile model on networks

- Start with a network
- Drop units of load randomly on nodes
- Each node has a threshold.
  Here = degree.
- Load on a node ≥ threshold
  ⇒ node topples, moves load to neighbors



Sandpile model on networks

- Start with a network
- Drop units of load randomly on nodes
- Each node has a threshold.
  Here = degree.
- Load on a node ≥ threshold
  ⇒ node topples, moves load to neighbors
- Neighbors may topple. Etc.
  Cascade (or avalanche) of topplings.



Power-law distribution of avalanche sizes, $P(s) \sim s^{-3/2}$

Power law tails (Universal behavior)

Avalance size follows power law distribution $P(s) \sim s^{-3/2}$

Power law tails seem to characterize the sizes of electrical blackouts, financial fluctuations, neuronal avalanches, earthquakes, landslides, overspill in water reservoirs, forest fires and solar flares.

[1] I. Dobson, B. A. Carreras, V. E. Lynch, and D. E. Newman, Chaos **17**, 026103 (2007).
[2] X. Gabaix, P. Gopikrishnan, V. Plerou, and H. E. Stanley, Nature **423**, 267 (2003).
[3] J. M. Beggs and D. Plenz, J, Neurosci. **23**, 11167 (2003).
[4] D. E. Juanico and C. Monterola, J. Phys. A **40**, 9297 (2007).
[5] T. Ribeiro, M. Copelli, F. Caixeta, and H. Belchior, PLoS ONE **5**, e14129 (2010).
[6] A. Saichev and D. Sornette, Phys. Rev. E **70**, 046123 (2004).
[7] S. Hergarten, Natural Hazards and Earth System Sciences **3**, 505 (2003).
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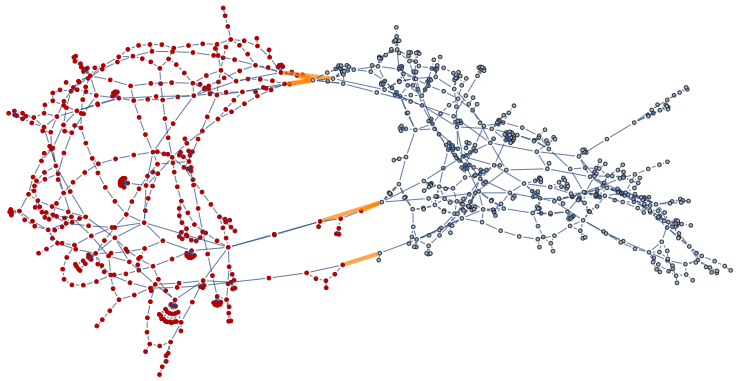
[9] P. Sinha-Ray and H. J. Jensen, Phys. Rev. E **62**, 3216 (2000).
[10] B. D. Malamud, G. Morein, and D. L. Turcotte, Science **281**, 1840 (1998).
[11] E. T. Lu and R. J. Hamilton, Astrophys. J. **380**, L89 (1991).
[12] M. Paczuski, S. Boettcher, and M. Baiesi, Phys. Rev. Lett. **95**, 181102 (2005).
[13] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).

BTW cascades on sparsely coupled networks

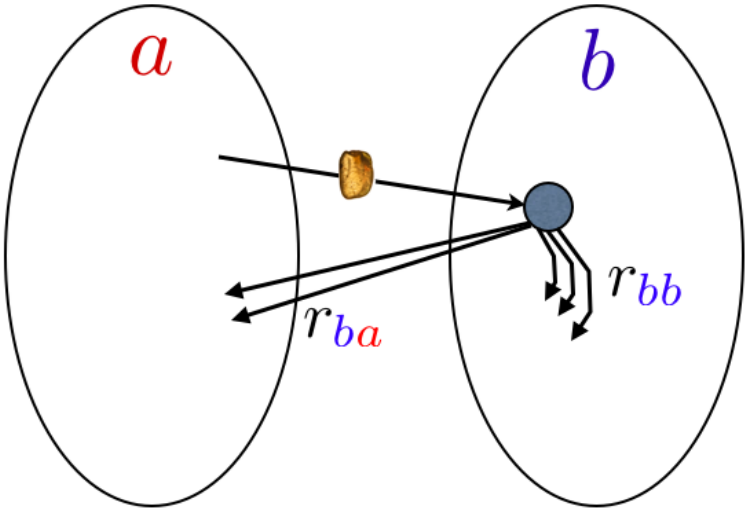
C. Brummitt, R. M. D'Souza, E. A. Leicht *PNAS* 109 (12), 2012.

Two-type network: a and b
Impact of increased a-b links.



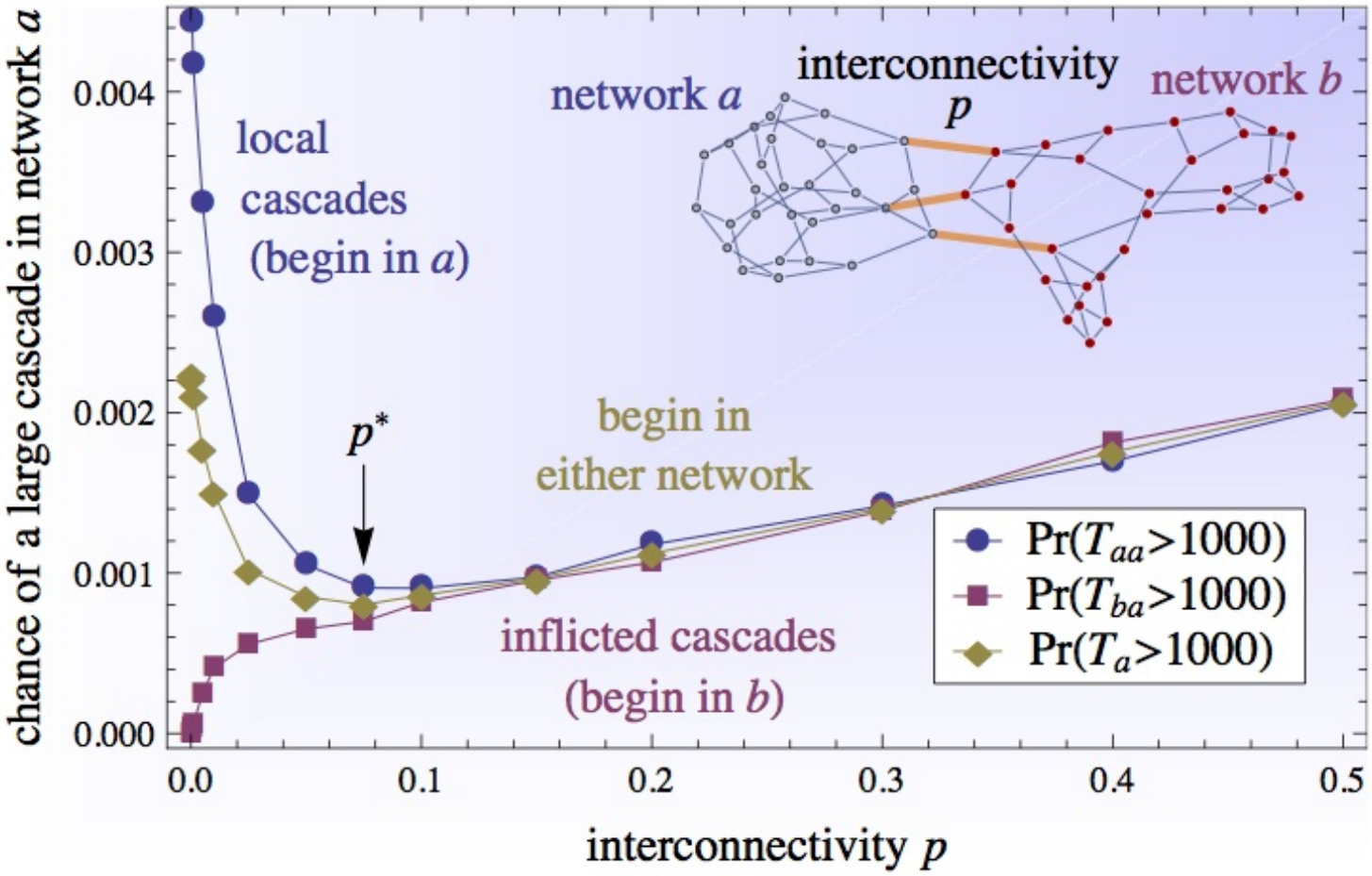
$p_a(k_a, k_b), p_b(k_a, k_b)$
(Configuration model)

Branching process treatment



$q_{ab}(r_{ba}, r_{bb}) :=$ the branch (children) distribution for an ab-*shedding*.

Main findings: For an individual network, optimal p^*

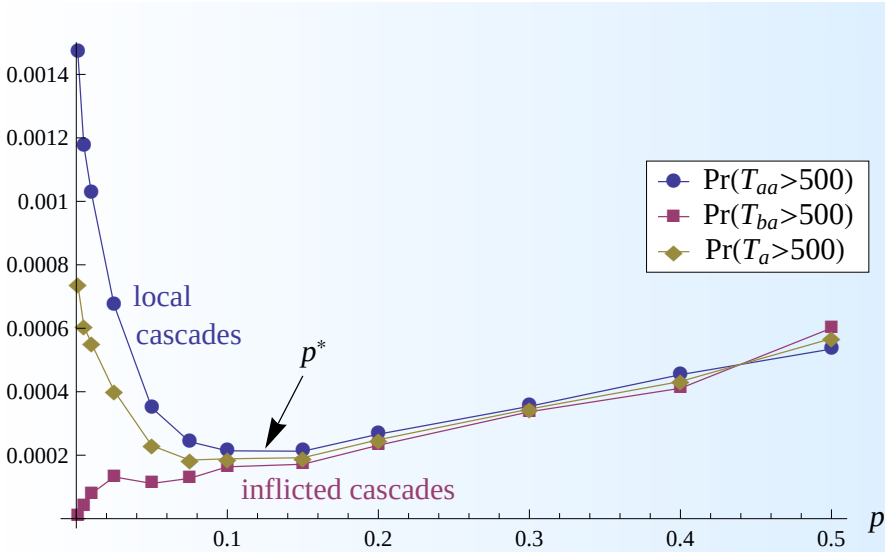


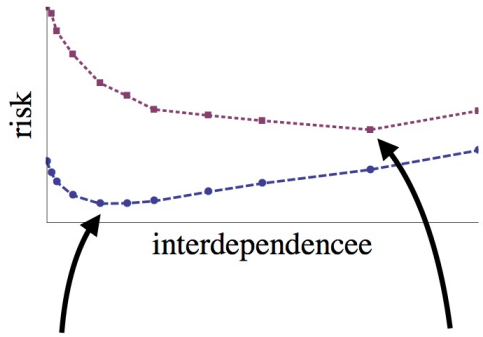
- (Blue curve) self-inflicted cascades (second network is reservoir).
  - (Red curve) inflicted from the second network
  - (Gold curve) Neglecting the origin of the cascade

Main findings: Increased systemic risk

- **More interconnections fuel larger system-wide cascades.**

– Each new interconnection adds capacity and load to the system (Here capacity is a node's degree, interconnections increase degree)



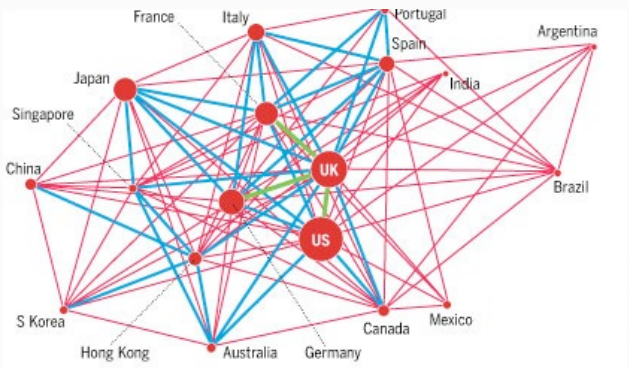


Frustrated equilibria

- So an individual operator adding edges to achieve p^* may inadvertantly cause larger global cascades.

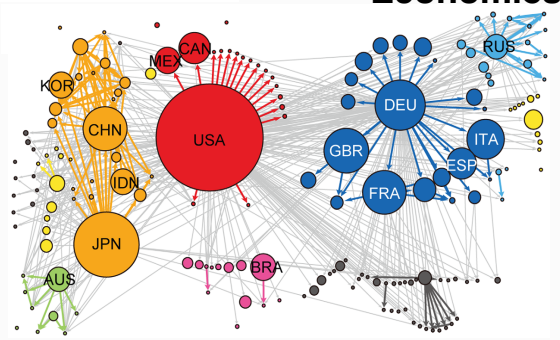
Optimal interdependence

"Some networking is good. Too much is overwhelming."



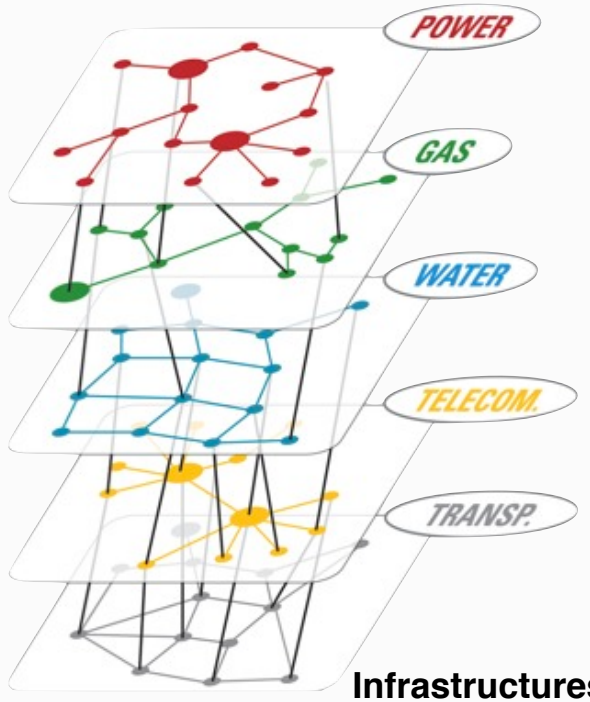
Andrew Haldane/Bank of England

Financial networks



Economies

Lee et al., PLoS ONE 2010

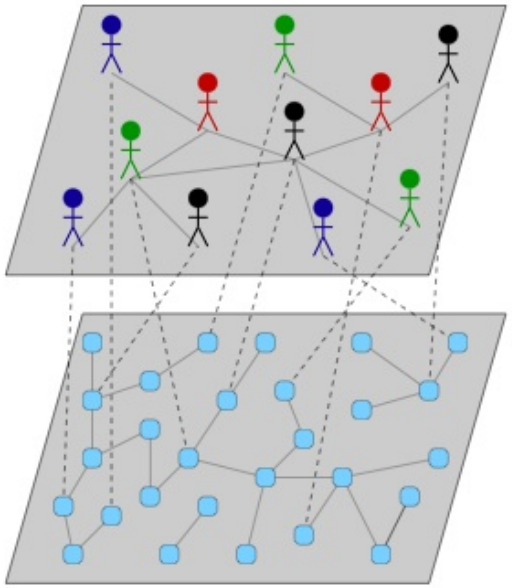


Infrastructures

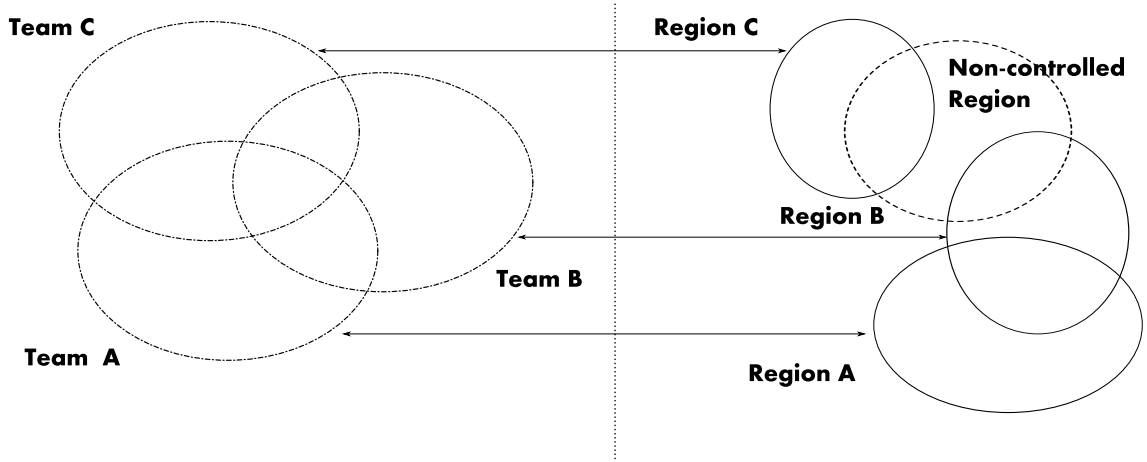
L. Dueñas-Osorio

- **Financial markets**: Battiston, et al., *J. of Econ. Dyn. & Control* 36 (2012).
- **Synchronization**: Hunt., Korniss, Szymanski. *PRL*, 2010.
- **"Islanding"** in power grids: Andersson, et al. *IEEE Trans. Power Systems*, 2005.
- **"Islanding"** among traders: Saavedra, Hagerty, Uzzi, *PNAS*, 2011, *PLoS ONE* 2011.

More interdependent networks – Socio-technical congruence



(a)

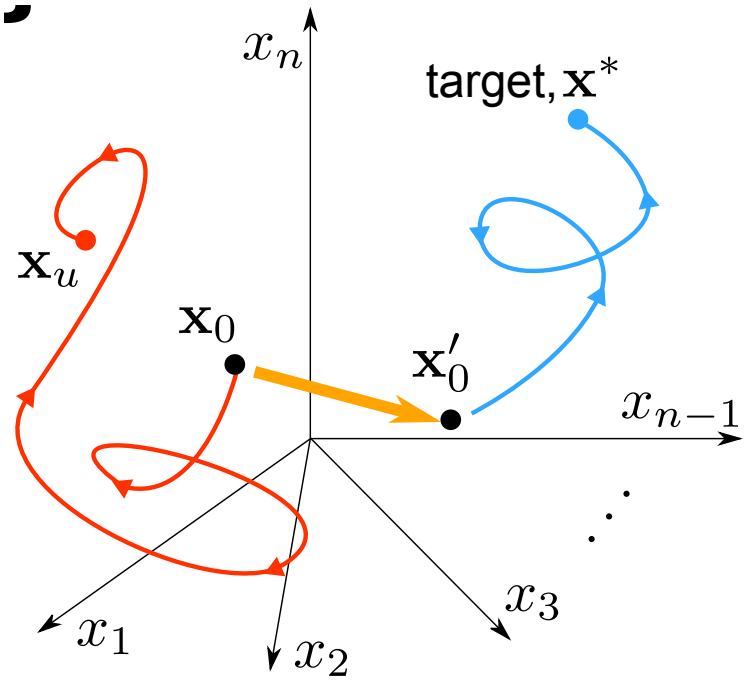


(b)

- (a) Open Source Software : Ecologies, team leaders, file owners, etc., affects on bug occurrence.
- (b) Modular teams & the modular technological systems
  Cooperative game theory / coalition games: incentivize informal cross-team collaboration: institutional memory

Controlling networks

A network with n nodes an n-dimensional phase space:



- Strong Control: want to control the state of each node.

- Weak/Partial Control: want to steer the system.

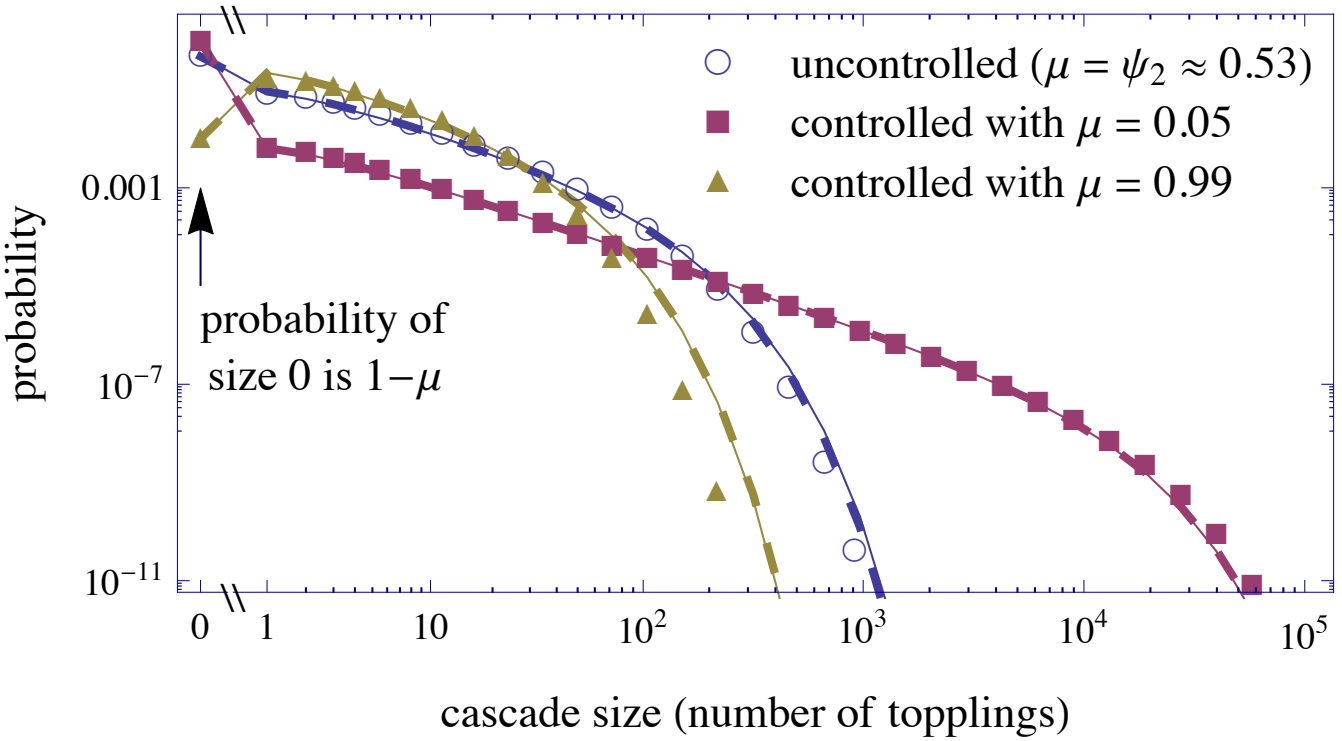
- Controlling self-organizing systems especially challenging: the system responds to the controller.

$\rightarrow$ Noël, Brummitt, R.D., Phys. Rev. Lett. **111**, 0780701 (2013).

Controlling the BTW model away from the SOC state

Noël, Brummitt, R.D., Phys. Rev. Lett. **111** *0780701, 2013*

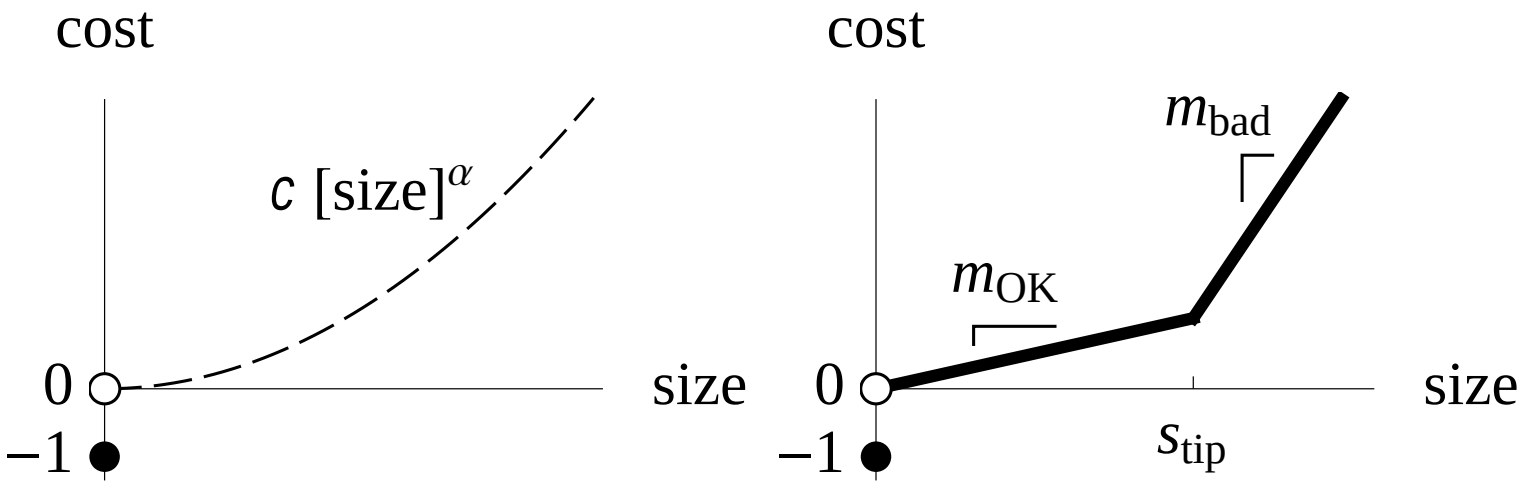
Control parameter μ: probability grain lands on a node at threshold



- Avoid cascades, $\mu = 0.05 \rightarrow$ larger cascades when they do occur.
- Ignite cascades, $\mu = 0.99 \rightarrow$ smaller cascades, but more frequent.

Accounting for cost of cascading events

Assuming larger events more costly.

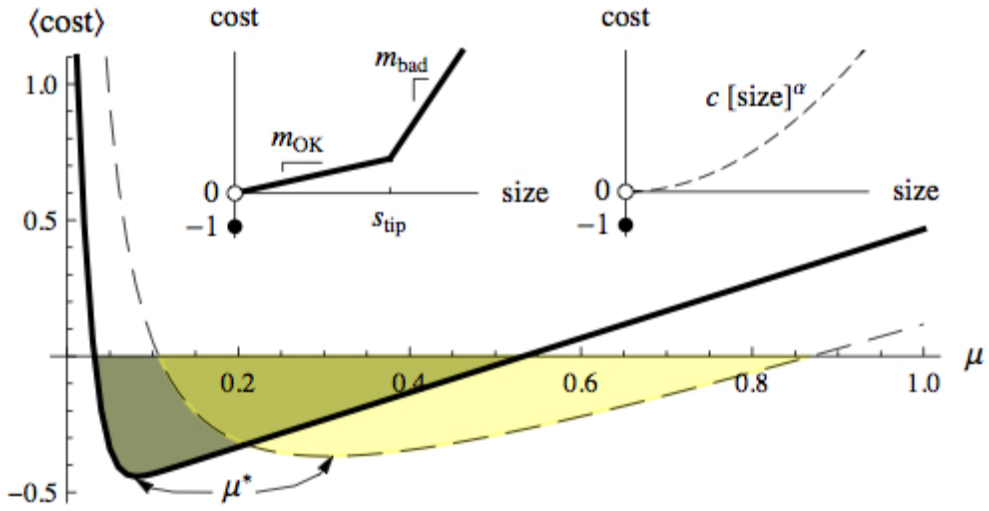


Accounting for costs: Optimal control levels

Too much control can be detrimental.

Accounting for costs with larger events more costly.

Optimal μ^*



- Frequently triggering cascades mitigates large events but sacrifices short-term profit.

- Avoiding cascades maximizes short-term profit but suffers from rare, massive events.

US DoD research initiative: Control of complex networks

FY2013 Multidisciplinary University Research Initiative (MURI)

"Predicting and Controlling Systems of Interdependent Networks: Exploiting Interdependence for Control"

$6.25million, 5 year project.

R. D. (PI), **Statistical Physics of Networks** , UC Davis
Jim Crutchfield, **Network Information Theory** , UC Davis
Leonardo Duenas-Osorio, **Critical Infrastructure** , Rice University
Jessica Flack, **Collective Computation** , University of Wisconsin
David C. Krakauer, **Emergence of order** , University of Wisconsin
Mehran Mesbahi, **Control Theory** , University of Washington
Michael Roukes, **NEMs oscillator arrays** , CalTech

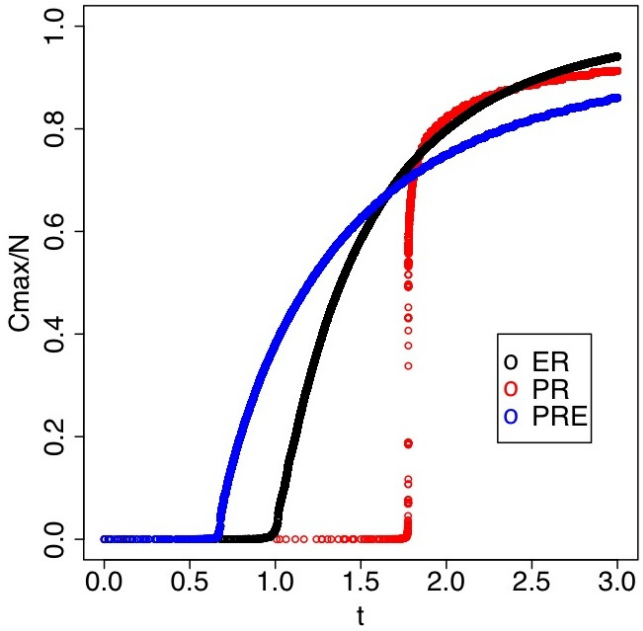
Conclusions

– Statistical physics enables (useful) models of real world networks

percolation, cascades, synchronization, epidemic spreading, ...

– Explosive Percolation as a model of extreme events

- Apply a patch repeatedly to delay onset of percolation
- Explosive onset when inevitable ultimately occurs
- Intervention impossible: Once detected, too late



– Optimal Interdependence

– Bridging control theory and stat phys in the context of networks

Recent Press & Acknowledgements

- "When Networks Network", *Science News*, Sept 22, 2012
- "The mathematics of averting the next big network failure", Wired, Mar 19, 2013.
- "Physicists get to grips with complex systems", Physics World, Aug 16, 2013.