Talent and experience in competitive social hierarchies

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Abstract

Hierarchy of social organization is a ubiquitous property of animal and human groups, linked to resource allocation, collective decisions, individual health, and even to social instability. Experimental evidence shows that both intrinsic abilities of individuals and social reinforcement processes impact the structure of hierarchy. We develop a rigorous model that incorporates both features and explore their synergistic effect on stability and the role of talent. For pairwise interactions, we show there is a trade-off between relationship stability and having the high ranks occupied by talented individuals. Extending this to open societies, where individuals may enter or leave the population, we show their are important societal effects that are a product of the interaction between talent and social processes, that cannot be observed if either effect dominates: (i) despite positive global correlation between talent and rank, paradoxically, local correlation is negative, and (ii) the removal of an individual can induce a series of rank reversals that can not be seen in traditional models that incorporate only social reinforcement. We show that the mechanism underlying the latter is the removal of an older individual of limited talent, who nonetheless was able to supress the rise of younger more talented individuals.

Introduction. Hierarchy is a central organizing principle of complex systems, manifesting itself in various forms in biological, social, and technological systems [1]. Therefore to understand complex systems, it is crucial to develop quantitative methods that describe hierarchies [2–5] and to identify their origins and benefits [6, 7]. Among the various forms of hierarchy, here we are concerned with social hierarchies emerging through competition. Such hierarchy represents a ranking of individuals based on social consensus: a high ranking individual is expected to win a conflict against a low ranking one. This type of organization is present in societies ranging from insects to primates and humans [3, 8–10], and has been linked to resource allocation, individual health, collective decisions, and social stability [7, 11–13].

The prevalence of social hierarchies motivated a long history of theoretical research in statistical physics and mathematical biology [6, 14–17]. The unifying theme in explaining the emergence of hierarchies is positive reinforcement of differences known as the winner (or looser) effect: initially equally ranked individuals repeatedly participate in pairwise competitions, and after an individual wins (or looses), the probability of him winning (or loosing) later competitions increases. The conditions for hierarchies to emerge and their structure was thoroughly investigated [9, 16–18].

A number of experimental studies investigated the role of intrinsic attributes and social reinforcement in hierarchy formation. These experiments focused on small groups of animals, and found that reinforcement and intrinsic differences affect hierarchy to varying extent depending on the context [18–20]. However, a general picture emerges: both abilities and experience contribute simultaneously to the rank of individuals. This observation seems to hold for species with relatively simple social interactions, such as cichlid fish [21], to species that form highly complex societies, such as primates [11, 22].

Despite the clear indication of experiments that both talent and reinforcement matter, we are lacking general theoretical understanding of their synergistic impact [23]. In this Letter, we develop a rigorous model incorporating both talent and social reinforcement and show that this captures a much richer landscape. For pairwise interactions, we show a trade-off between societal stability and having more talented individuals as the high-ranked leaders. We then extend the model to open populations, where individuals enter and leave the group, and we characterize both the global and the local structure of the hierarchies. Another pressing issue is to understand the hierarchy's response to perturbation, e.g., the effect of removing an individual. In particular, animal behavior experts must often make strategic decisions to remove individuals from captive societies due to health issues or in attempt to promote social stability, which sometimes lead to unanticipated large instabilities [12, 24]. We show that traditional models predict smooth response and no rearrangement; if, however, both talent and social reinforcement are equally important, removal of an individual can lead to a non-trivial series of rank reversals.

Model. Our starting point is a variant of the classic Bonabeau model [6]. It describes a social group with N members, where the rank of each member is determined by its ability to defeat others in pairwise competitions. This time-dependent ability is quantified by a score $x_i(t)$, where the subscript indexes the individuals. The scores are initially identical ($x_i(t=0) \equiv 0$) and they change through two discrete-time processes. First, through positive feedback of differences: In each time step, participants are randomly paired to compete with each other, and the winner increases its score by δ. Individual i wins against j with probability

$$Q_{ij}(t) = \frac{1}{1+\exp[-\beta(x_i(t)-x_j(t))]}, \tag{1}$$

where β is an inverse temperature-like parameter, for large β the outcome of the fight is deterministic, for $\beta = 0$ both parties have equal chance to win irrespective of their score. The second process is forgetting: The effect of a fight wears off exponentially, i.e., $x_i(t)$ is reduced by $\mu x_i(t)$ ($0 \leq \mu \leq 1$) in each time step. Approximating the process with the deterministic equation

$$x_i(t+1) = (1-\mu)x_i(t) + \frac{1}{N}\delta \sum_{j \neq i} Q_{ij}(t), \tag{2}$$

it was shown that depending on the relative strength of reinforcement and decay the model supports either egalitarian ($x_i \equiv 0$) or hierarchical ($x_i \not\equiv 0$) steady state solutions [6, 25].

To include intrinsic attributes or talents, we offset the score of each participant in Eq. (1) by base intrinsic abilities b_i and b_j:

$$Q_{ij}(t) = \frac{1}{1+\exp[-\beta(x_i(t)+b_i-x_j(t)-b_j)]}. \tag{3}$$

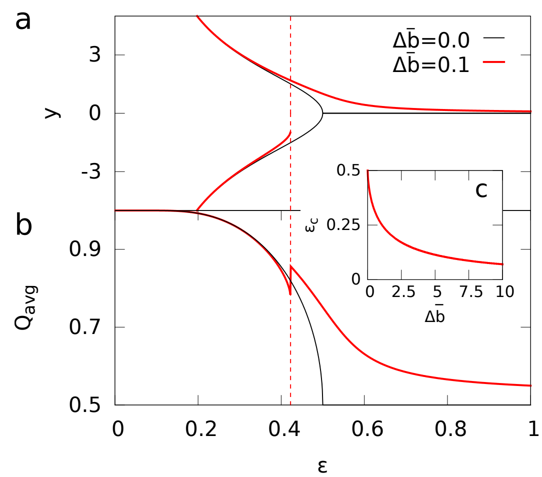

FIG. 1. **Fairness and stability** ($N = 2$)**. (a)** Score difference in function of ϵ, without (black) and with intrinsic difference (red). If $\Delta b \neq 0$, for large ϵ only one hierarchical solution exists corresponding to the fair ranking, i.e., rank is determined by talent; and through a discontinuous transition a new solution emerges corresponding to the opposite, unfair ordering. **(b)** We quantify the stability of the dominant-subordinate relationship with $Q_{\rm avg}$, strong reinforcement leads to stability. **(c)** Critical point in function of $\bar{\Delta b}$.

The parameters b_i and b_j quantify attributes that are independent of social processes on the timescale of hierarchy formation, yet are relevant to conflict outcomes. Examples may include strength, intelligence, or other talents.

Two individuals. To understand the consequences of intrinsic differences, it is insightful to first investigate a population of $N = 2$ individuals. The deterministic equation describing the steady state is provided by

$$0 = -\mu \Delta x + \delta \left(\frac{2}{1 + \exp\left[-\beta(\Delta x + \Delta b)\right]} - 1 \right), \tag{4}$$

where $\Delta x = x_1 - x_2$ and $\Delta b = b_1 - b_2$. Introducing $y = \beta \Delta x$, $\bar{\Delta b} = \beta \Delta b$ and $\epsilon = \mu/(\delta\beta)$ leads to

$$0 = -\epsilon y + \frac{2}{1 + \exp\left[-y - \bar{\Delta b}\right]} - 1, \tag{5}$$

meaning that the steady state is determined by the talent difference and a single parameter ϵ measuring the relative strength of decay to social reinforcement [26].

Systematically changing ϵ, we observe a transition at ϵ_c separating regimes with one and two stable solutions, the nature of the transition depends on the presence of intrinsic differences. If $\bar{\Delta b} = 0$ (Fig. 1a black line), we recover the original model: For $\epsilon > \epsilon_c^{(0)}$ we have one solution, representing the egalitarian state $y = 0$, and at ϵ_c two symmetric hierarchical solutions ($y_1 = -y_2 \neq 0$) emerge through a pitchfork bifurcation. If $\bar{\Delta b} \neq 0$ (Fig. 1a red line): For $\epsilon > \epsilon_c$ we again find just one solution; this solution, however, is not egalitarian ($y > 0$), the more talented outranks the less talented. At ϵ_c a new stable solution appears through a discontinuous transition supporting the opposite order, the less talented outranking the more talented. In other words, social reinforcement can outpace intrinsic differences. We call the $y > 0$ solution fair and the $y < 0$ one unfair, since the more talented as higher ranked has better access to resources, potentially improve collective decisions, and has higher chance to foster offspring.

Figure 1c shows ϵ_c dependent on $\bar{\Delta b}$. In general, no closed-form solution is available; limiting cases, however, can be worked out: for small differences we find $(\epsilon_c - 1/2) \sim \bar{\Delta b}^{2/3}$ and for large differences $\epsilon_c = \bar{\Delta b}^{-1}$. The latter indicates that increasing talent difference or decreasing reinforcement push the system to a regime where only the fair solution exists. This prompts the question: what does the system benefit from the social reinforcement process?

To answer this question, we quantify the stability of a dominant-subordinate relationship with Q_{avg}, the probability that the dominant wins a conflict averaged over the stable steady state solutions, $Q_{\text{avg}} \approx 1/2$ indicating an unstable, $Q_{\text{avg}} \approx 1$ a well-defined relationship. Figure 1b shows that increasing the weight of social reinforcement increases Q_{avg}, revealing a fundamental trade-off between stability and fairness: stable relationships require strong social reinforcement; however, strong reinforcement allows for unfair hierarchical states. This provides rationale for why hierarchies would be influenced equally by both intrinsic talent and social processes.

Open populations. So far we focused on the relationship of two individuals, now we turn our attention to larger, changing populations. We study groups of N individuals where the talent of each individual is drawn randomly from a distribution $p(b)$. We initially allow the population to reach a stable ranking. Then in each step t, we remove a random individual and add a new member i to the bottom of the society, i.e., $x_i(t) = 0$, and again allow the population to reach a stable ranking. For simplicity we restrict our investigation to the $\beta \to \infty$ limit, in which case Q_{ij} becomes a step function: i always wins if i outranks j. This allows us to explicitly formulate the condition for two consecutively ranked individuals to reverse ranks as

$$b(k+1) - b(k) > \Delta x \equiv \frac{\delta}{\mu(N-1)}, \tag{6}$$

where $b(k)$ is the talent of the individual ranked kth ($k = 1$ is the top and $k = N$ is the bottom rank) and $\Delta x = x(k) - x(k+1)$ doesn't depend on k. Therefore, instead of simulating dynamics of Eq. (2), we check each consecutive pair and if Eq. (6) is satisfied, we reverse their order. We repeat this until no more change is found.

The talent b of an individual represents an intrinsic ability or a combination of abilities that influence the outcome of a fight. Here we investigate the case where $p(b)$ is Gaussian. Indeed, body size, intelligence, and other relevant abilities are often normally distributed. Equation (6) only depends on the difference of abilities, allowing us to shift the mean of $p(b)$ arbitrarily; furthermore, we can set its variance by rescaling Δx. Therefore, without further loss of generality, we restrict our investigation to the standard normal distribution.

We now systematically investigate the structure of the emergent hierarchy in function of Δx through simulations and analytical calculations (details in the Supplementary Information). We measure correlation between rank and talent (τ_{tal}) and between rank and experience (τ_{exp}) using Kendall's tau coefficient, where experience is the amount of time an individual spent in the population. For example, $\tau_{\text{tal}} = 1$ indicates talent completely determines rank and $\tau_{\text{tal}} = 0$ indicates no correlation. Figure 2a shows that for large Δx rank is dominated by experience, meaning that the only way to advance in hierarchy is if a higher ranking individual is removed; and for small Δx rank is dominated by talent. These two limiting cases are separated by a regime where both talent and experience matter, the crossover point where $\tau_{\text{tal}} = \tau_{\text{exp}} = 1/2$ is $\Delta x^* \approx 0.36$.

Many experiments showed partial correlation between rank and certain individual attributes [9]; this, however, can be explained by incorrect identification of the relevant abilities. Some experiments were designed to explicitly demonstrate that both abilities and social history matter by showing that hierarchies are only partially re-established after separating

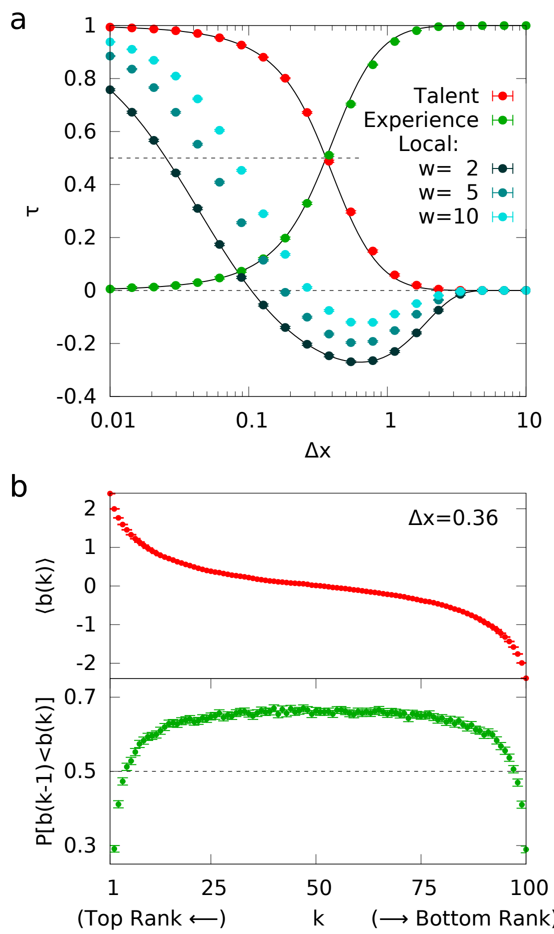

FIG. 2. **Talent-rank correlation. (a)** Kendall's tau in function of Δx. Global talent-rank (red) and experience-rank (green) correlation shows a crossover between talent and experience dominated limiting cases. Counter-intuitively, we find that locally talent and rank are anti-correlated (blues). **(b)** Local rank-talent anti-correlation. In the crossover regime, the expected talent increases with rank (red), yet the probability that an individual's immediate superior is less talented is greater than 1/2 (green). Results are shown for populations of $N = 100$, continuous lines are analytical solutions [RRR,SI]. Data points are an average of 10,000 independent samples and error bars represent the 95% CI.

then re-joining animal groups [21]; yet, without knowledge of relevant attributes the value of talent-rank correlation remains unknown. In the Supplemetary Information, we show that

$\tau_{\text{tal}}+\tau_{\text{exp}} = 1$ if talent and introduction time are independent. And while relevant abilities are challenging to identify, τ_{exp} is straight forward to measure. Indeed, Tung et al. created groups of rhesus macaques by introducing animals one-by-one into an eclosure and found that the Spearman's correlation between rank and experience is $\rho_{\text{exp}} = 0.61$, providing evidence that some real systems are in the cross-over regime [11].

In addition to global correlations, we also measure local orderedness by calculating $\tau_{\text{tal}}(w)$, the talent-rank correlation averaged over a sliding window of length w. Counter-intuitively, Fig. 2a shows that in the crossover regime $\tau_{\text{tal}}(w)$ is negative, meaning that locally rank and talent are anti-correlated. Figure 2b provides an additional aspect of this paradox situation: The expected talent $\langle b(k)\rangle$ of an individual ranked kth at a random time step monotonically increases with rank; yet the probability that the $(k-1)$th individual, the one immediately outranking the kth, is less talented than the kth is greater than 1/2.

To understand the mechanism producing the local anti-correlation, first consider two consecutive individuals forming an ordered pair with respect to talent, i.e., $b(k) < b(k-1)$. If a new individual arrives with talent b such that $b(k)+\Delta x < b$ and $b(k-1) < b < b(k-1)+\Delta x$, it can pass the kth individual, but cannot pass the $(k-1)$th, lodging itself between the two and creating an unordered pair. If, however, the pair is unordered, i.e., $b(k) > b(k-1)$, any individual passing the kth individual will necessarily pass $(k-1)$th too. Therefore an unordered pair will remain unordered until one of the pair is removed. This asymmetry in creating ordered and unordered pairs is responsible for the local anti-correlation.

Finally, we also investigate the effect of removing an individual. We find that in the talent or experience dominated limiting cases the system's response is trivial and no re-organization happens. However, Figure 3 shows that the probability that removal induces rank reversals p_{rr} is non-zero in the intermediate regime. And both p_{rr} and the average number of these rank reversals N_{diff} peak near, but not exactly at, the crossover point Δx^*. For induced rank reversals to happen, three consecutively ranked individuals are needed in opposite order with respect to talent, i.e. $b(k+1) > b(k) > b(k-1)$. If the condition $b(k+1) - b(k-1) > \Delta x$ is satisfied, the removal of the kth individual allows the $(k+1)$th to pass the $(k-1)$th. In other words, the kth individual is not talented enough to further advance in society, but is capable of holding back a more talented contender. Understanding the response of hierarchies to

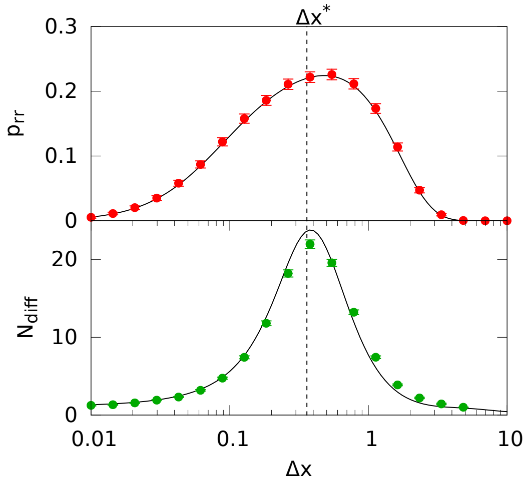

FIG. 3. **Removal of a group member.** In the limiting talent and age dominated cases removal has trivial effect, while in the crossover regime the removal of an individual causes rank reversals with finite probability (red). The average number of rank reversals peaks near Δx^* (green). Results are shown for populations of $N = 100$, continuous lines are analytical solutions [RRR,SI]. Data points are an average of 50,000 time steps and error bars represent the 95% CI.

external perturbation is an important issue. Particularly, removal of animals from primate groups can sometimes lead to unanticipated large instabilities [12, 24]. Here we demonstrated that traditional models of hierarchy formation only considering either intrinsic differences or social feedback cannot explain such rank events, and that both effects have to be present simultaneously.

Discussion. In this Letter, we studied the synergistic effect of intrinsic differences and social reinforcement on the structure of competitive social hierarchies, and we identified behaviors that cannot be observed if either effect dominates. We derived our model assuming random pairwise conflicts and a winner effect, we believe that the results can be interpreted more generally: (i) The mechanism behind local talent-rank anti-correlation and removal induced rank reversals is that to pass someone in rank it is not enough to be more talented, but the talent difference has to be sufficient to compensate for the advantage of being higher ranked – a process that we believe is relevant to a wide range of other rankings, examples might include bibliometric rankings of scientists, best seller lists, or sports rankings. (ii) We introduced parameter b to capture individual abilities such as body weight or strength; however, it can be though of as a proxy for support of kin or as a simplified model of reputation received in exchange for non-adversarial social interactions.

Finally, our results prompt many open research questions, both experimental and theoretical. Local anti-correlation and removal induced rank reversals are predictions that are testable through experiments or observational data. We are also obliged to acknowledge that the model does not capture the full complexity of real systems, for example, future work may investigate the role of ageing, e.g., slow deterioration of talent; non-normal distribution of talent; or non-linear hierarchies, where a social tier might be occupied by multiple individuals.

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[1] A. Zafeiris and T. Vicsek, (2017), arXiv:1707.01744.

[2] A. Trusina, S. Maslov, P. Minnhagen, and K. Sneppen, Phys. Rev. Lett. **92**, 178702 (2004), arXiv:0308339 [cond-mat].

[3] H. Fushing, M. P. McAssey, B. Beisner, and B. McCowan, PLoS One **6**, 1 (2011).

[4] E. Mones, L. Vicsek, and T. Vicsek, PLoS One **7** (2012), 10.1371/journal.pone.0033799.

[5] C. De Bacco, D. B. Larremore, and C. Moore, (2017), arXiv:1709.09002.

[6] E. Bonabeau, G. Theraulaz, and J.-L. Deneubourg, Phys. A Stat. Mech. its Appl. **217**, 373 (1995).

[7] T. Nepusz and T. Vicsek, PLoS One **8** (2013), 10.1371/journal.pone.0081449, arXiv:1308.0029.

[8] I. D. Chase, Am. Sociol. Rev. **45**, 905 (1980).

[9] M. Mesterton-Gibbons, Y. Dai, and M. Goubault, Math. Biosci. **274**, 33 (2016).

[10] J. Lerner and A. Lomi, Appl. Netw. Sci. **2**, 24 (2017).

[11] J. Tung, L. B. Barreiro, Z. P. Johnson, K. D. Hansen, V. Michopoulos, D. Toufexis, K. Michelini, M. E. Wilson, and Y. Gilad, Proc. Natl. Acad. Sci. **109**, 6490 (2012), arXiv:arXiv:1011.1669v3.

[12] B. A. Beisner, J. Jin, H. Fushing, and B. McCowan, Curr. Zool. **61**, 70 (2015), arXiv:15334406.

[13] J. J. Vandeleest, B. A. Beisner, D. L. Hannibal, A. C. Nathman, J. P. Capitanio, F. Hsieh, E. R. Atwill, and B. McCowan, PeerJ **4**, e2394 (2016).

[14] H. G. Landau, Bull. Math. Biophys. **13**, 1 (1951).

[15] I. D. Chase, Syst. Res. Behav. Sci. **19**, 374 (1974).

[16] E. Ben-Naim and S. Redner, J. Stat. Mech. Theory Exp. **2005**, 4 (2005), arXiv:0503451 [cond-mat].

[17] C. Castellano, S. Fortunato, and V. Loreto, Rev. Mod. Phys. **81**, 591 (2009).

[18] Y. Hsu, R. L. Earley, and L. L. Wolf, Biol. Rev. **81**, 33 (2005).

[19] L. A. Dugatkin, M. S. Alfieri, and A. J. Moore, Ethology **97**, 94 (1994).

[20] J. P. Beaugrand and P. A. Cotnoir, Behav. Processes **38**, 287 (1996).

[21] I. D. Chase, C. Tovey, D. Spangler-Martin, and M. Manfredonia, Proc. Natl. Acad. Sci. **99**, 5744 (2002).

[22] S. K. Seil, D. L. Hannibal, B. A. Beisner, and B. McCowan, Am. J. Phys. Anthropol. (2017), 10.1002/ajpa.23296.

[23] J. L. Beacham, Behaviour **140**, 1275 (2003).

[24] B. McCowan, K. Anderson, A. Heagarty, and A. Cameron, Appl. Anim. Behav. Sci. **109**, 396 (2008).

[25] L. Lacasa and B. Luque, Phys. A Stat. Mech. its Appl. **366**, 472 (2006).

[26] It is interesting to note that introducing $m = \mu\Delta x$ and $\tilde{\beta} = \beta\delta/(2\mu)$ maps Eq. (4) to the meanfield Ising model and the intrinsic difference maps to a non-zero external field $h = \mu\Delta b/\delta$.