1) Show that \( \neg(p \lor \neg q) \) and \( q \land \neg p \) are equivalent.

2) Let \( p, q, r \) be the following propositions:
\( p : \) You get an A in this class
\( q : \) You understand every exercise in the book
\( r : \) You like logic puzzles

Translate the statements below into propositional logic.

a) You like logic puzzles if you do every exercise in the book.
b) For you to get an A in the class it is sufficient that you understand every exercise in the book.
c) For you to get an A in this class it is necessary for you to like logic puzzles.
d) You get an A in this class even though you dislike logic puzzles or you do not understand every exercise in the book.

3) Using De Morgan’s laws to show \( \neg(p \land q) \land p \rightarrow \neg q \).

4) Prove that if \( n^2 \) is an even integer, \( n \) is an even integer. What type of proof technique did you use?

5) State the rules of modus ponens and modus tollens for an implication.

6) Prove via contradiction that if you consider 8 days at least 2 of them must fall on the same day of the week.

7) Prove there exists a number \( n \) such that \( n \) is the product of two prime numbers. Did you use a constructive or nonconstructive existence proof?

8) Prove the set identity \( (A \cup B) = A \cap B \) by showing each resulting set is a subset of the other. (i.e. Prove if \( x \in (A \cup B) \leftrightarrow x \in A \cap B \))

9) Are the following implications/arguments valid? If they are invalid, what is the reason?
a) If today is Thursday then \( 1+1=3 \).
b) If today is Thursday, then I will go to the beach. I am at the beach so today is Thursday.
c) There exists a student who likes discrete math. John is a student therefore John likes discrete math.
d) There does not exist a student who likes doing homework, therefore all students dislike doing homework.

10) Prove the $\sqrt{2}$ is irrational. *(Note, I would give you intermediate steps and walk you through this proof.)*

11) Are the following bijections? If not, explain why.
a) $f : \mathbb{Z} \to \mathbb{R}, f(x) = 1/x$.
a) $f : \mathbb{Z}^+ \to \mathbb{R}, f(x) = 1/x$.

12) Evaluate $\sum_{i=2}^{n} i$ where $n$ is an odd number.

13) Prove that if $n$ is an integer, $n = \lfloor n/2 \rfloor + \lceil n/2 \rceil$.

14) Translate the following numbers into binary format:
a) 64
b) 128
c) 1028
d) 133

15) What is the power set of the set $A = \{1, 2, 3\}$. What is the cardinality of the power set of a general set $S$?