Sequences and Summations

1. A sequence is a discrete structure used to represent an ordered list. Formally, a sequence is a function from a subset of the set of integers to a set $S : n \mapsto a_n$.

   Examples:
   - Sequence $\{a_n\} = \{1/n\}$ for $n = 1, 2, 3, \ldots$
   - Sequence $\{(-1)^n\}$ for $n = 1, 2, 3, \ldots$

2. An arithmetic progression is a sequence of the form
   $$a, a + d, a + 2d, a + 3d, \ldots, a + nd, \ldots$$

   Example:
   - The sequence $\{1, 3, 5, 7, 9, \ldots\} = \{(1 + 2n), n = 1, 2, 3, \ldots\}$.

3. A geometric progression is a sequence of the form
   $$a, ar, ar^2, ar^3, \ldots, ar^n, \ldots$$

   Example:
   - The sequence $\{1, -1, 1, -1, 1, \ldots\} = \{a_n = (-1)^n, n = 0, 1, 2, \ldots\}$.

4. Summation:
   $$a_m + a_{m+1} + \cdots + a_n = \sum_{j=m}^{n} a_j.$$

   Examples:
   - $\sum_{j=1}^{5} j^2 = \sum_{s \in \{0,2,4\}} s^2 = \ldots$

5. Two frequently used summation formulae:
   $$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$
   $$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

   We will prove these identities by mathematical induction later.
6. The sum of terms of a geometric progression is given by

\[ S_n = \sum_{j=0}^{n} ar^j = \begin{cases} \frac{a(r^{n+1}-1)}{r-1} & \text{if } r \neq 1 \\ (n+1)a & \text{if } r = 1 \end{cases} \]

7. Question: the sum of terms of arithmetic progression is given by

\[ A_n = \sum_{j=1}^{n}(a + jd) = \]

8. A note on binary notation (representing integers at bit strings).

Example binary number 010001110.

f: \(\mathbb{Z} \rightarrow B\)

\[
\begin{align*}
    f(0) &= 0 \\
    f(1) &= 1 \\
    f(2) &= 10 \\
    f(3) &= 11 \\
    f(4) &= 100 \\
    f(5) &= 101 \\
    f(6) &= 110 \\
    f(7) &= 111 \\
    f(8) &= 1000 \\
    f(9) &= 1001 \\
    \ldots
\end{align*}
\]