Sequences and Summations

1. A sequence is a discrete structure used to represent an ordered list. Formally, a **sequence** is a function from a subset of the set of integers to a set $S : n \mapsto a_n$.
   Examples:
   - Sequence $\{a_n\} = \{1/n\}$ for $n = 1, 2, 3, \ldots$
   - Sequence $\{(-1)^n\}$ for $n = 1, 2, 3, \ldots$

2. An *arithmetic progression* is a sequence of the form
   $$a, a + d, a + 2d, a + 3d, \ldots, a + nd, \ldots$$
   Example:
   - The sequence $\{1, 3, 5, 7, 9, \ldots\} = \{(1 + 2n), n = 1, 2, 3, \ldots\}$.

3. A *geometric progression* is a sequence of the form
   $$a, ar, ar^2, ar^3, \ldots, ar^n, \ldots$$
   Example:
   - The sequence $\{1, -1, 1, -1, 1, -1, \ldots\} = \{a_n = (-1)^n, n = 0, 1, 2, \ldots\}$.

4. Summation:
   $$a_m + a_{m+1} + \cdots + a_n = \sum_{j=m}^{n} a_j.$$
   Examples:
   - $\sum_{j=1}^{5} j^2 =$
   - $\sum_{s \in \{0,2,4\}} s^2 =$

5. Two frequently used summation formulae:
   $$\begin{aligned} \sum_{j=1}^{n} j &= \frac{n(n+1)}{2} \\ \sum_{j=1}^{n} j^2 &= \frac{n(n+1)(2n+1)}{6} \end{aligned}$$
   We will prove these identities by mathematical induction later.

6. The sum of terms of a geometric progression is given by

$$S_n = \sum_{j=0}^{n} ar^j = \begin{cases} \frac{a(r^{n+1}-1)}{r-1} & \text{if} \quad r \neq 1 \\ (n+1)a & \text{if} \quad r = 1 \end{cases}$$

7. Question: the sum of terms of arithmetic progression is given by
   $A_n = \sum_{j=1}^{n}(a + jd) =$

8. A note on binary notation (representing integers at bit strings).

   Example binary number 010001110.

   f: $\mathbb{Z} \to B$
   f(0) = 0
   f(1) = 1
   f(2) = 10
   f(3) = 11
   f(4) = 100
   f(5) = 101
   f(6) = 110
   f(7) = 111
   f(8) = 1000
   f(9) = 1001
   ...