1. A set is an unordered collection of distinct objects. Notation \( A, B, C, \ldots \). Duplicate elements only count once, e.g. if \( a = b \) then \( A = \{a,b,c\} = \{a,c\} \).

2. The objects in a set are also called the elements, or members, of the set. A set is said to contain its elements. We denote that \( x \) is an element of \( A \) by \( x \in A \).

3. Set builder notation \( A = \{x | P(x)\} \), \( A \) is the set of all \( x \) such that \( P(x) \) holds.

4. Two sets \( A \) and \( B \) are equal if and only if they have the same elements.
   \[ A = B \iff \forall x (x \in A \iff x \in B). \]

5. The set \( A \) is a subset of \( B \) iff (“if and only if”) every element \( A \) is also an element of \( B \).
   \[ A \subseteq B \iff \forall x(x \in A \rightarrow x \in B). \]
   If it is a proper subset: \( A \subset B \) if \( \forall x(x \in A \rightarrow x \in B) \land \exists x (x \in B \land x \notin A) \).

6. The empty (null) set contains no elements and is denoted \( \emptyset \) or \{\}.

7. For every set \( S \), \( \emptyset \subset S \) and \( S \subseteq S \).

8. Sets can be composed of sets, for example \( S = \{\{A\}, \{B\}\} \). (Note, \( S \notin S \), but \( S \in \{S\} \).)
   For instance, \( \{\emptyset\} \) (also denoted \( \{\{\}\}\) ) is the set containing the empty set, so \( \{\emptyset\} \notin \emptyset \), but \( \{\emptyset\} \in \{\{\emptyset\}\} \).

9. The universal set \( U \) contains all objects under consideration (all \( x \) in the domain \( U \)).

10. Venn diagram.

11. If there are \( n \) distinct elements in the set \( S \) where \( n \) is a nonnegative integer, we say that \( S \) is a finite set. \( n = |S| \) is the cardinality of \( S \). A set is said to be infinite if it is not finite.

12. Given a set \( S \), the power set of \( S \) is the set of all subsets of \( S \), denoted \( P(S) \). e.g., let \( S = \{1,2,3\} \), then \( P(S) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\} \). The cardinality \( |P(S)| = 2^{|S|} \) for \(|S| \) finite.

13. The ordered \( n \)-tuple \((a_1, a_2, \ldots, a_n)\) is the ordered collection that has \( a_1 \) as its first element, \( a_2 \) as its second element, \ldots, and \( a_n \) as its \( n \)th element. Here duplicates matter! \( n = 2, 2 \)-tuples are called ordered pairs.
14. Let \( A \) and \( B \) be sets. The **Cartesian product** of \( A \) and \( B \), denoted \( A \times B \), is the set

\[
A \times B = \{(a, b) \mid a \in A \land b \in B\}.
\]

e.g., The Cartesian product \( A \times B \) of \( A = \{1, 2\} \) and \( B = \{a, b, c\} \).

\[
A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}.
\]

These are *ordered sets* so \( A \times B \neq B \times A \) (unless \( A = B \), or \( A = \emptyset \), or \( B = \emptyset \)).

15. The Cartesian product of the sets \( A_1, A_2, \ldots, A_n \):

\[
A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \ldots, n\}.
\]

16. **Universal quantifier:** \( \forall x \in S(P(x)) \) means \( \forall x (x \in S \rightarrow P(x)) \).

   **Existential quantifier:** \( \exists x \in S(P(x)) \) means \( \exists x (x \in S \land P(x)) \).

   **Truth set:** If \( \forall x P(x) \), the *truth set* of \( P(x) \) is the set \( U \).

   If \( \exists x P(x) \), the *truth set* of \( P(x) \) is nonempty.

17. The **union** of \( A \) and \( B \):

\[
A \cup B = \{x \mid x \in A \lor x \in B\}.
\]

18. The **intersection** of \( A \) and \( B \):

\[
A \cap B = \{x \mid x \in A \land x \in B\}.
\]

19. \(|A \cup B| = |A| + |B| - |A \cap B| \) (principles of inclusion-exclusion).

20. Two sets are called **disjoint** if their intersection is the empty set \( \emptyset \).

21. The **difference** of \( A \) and \( B \): \( A - B = \{x \mid x \in A \land x \notin B\} \).

   \( A - B \) is also called the *complement* of \( B \) with respect to \( A \).

22. The complement of the set \( A \): \( \overline{A} = \{x \mid x \notin A\} \).

23. Set identities, refer to Table 1 in section 2.2.

   Example: Proof of De Morgan’s law: \( \overline{A \cap B} = \overline{A} \cup \overline{B} \).

24. Generalization union: \( A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i \).

   Generalization intersection: \( A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i \).

25. Reading: computer representation of sets.