1. A function from $A$ to $B$ is an assignment of exactly one element of $B$ to each element of $A$.
   Notation: $f : A \rightarrow B$, $f(a) = b$
   $A$ is called the domain of $F$ and $B$ is the codomain.
   The set $\{f(a) \mid a \in A\}$ is called the range of $f$.

2. $f : A \rightarrow B$ is one-to-one or injective if $f(a) = f(b)$ then $a = b$.
   (If not injective it is many-to-one.)
   Examples:
   - Is the function $f : \mathbb{Z} \rightarrow \mathbb{Z}$, $f(x) = x^2$ one-to-one?
   - Is the function $f : \{\text{US residents}\} \rightarrow \mathbb{Z}$, $f(x) = \text{SSN}$ one-to-one?

3. $f : A \rightarrow B$ is onto, or surjective, if for any $b \in B$, there is an $a \in A$ with $f(a) = b$.
   Examples:
   - Is the function $f : \mathbb{Z} \rightarrow \mathbb{Z}$, $f(x) = x^2$ onto? (Is every $\mathbb{Z}$ the square of a number?)
   - Is the function $f(x) = x + 1$ from $\mathbb{Z}$ to $\mathbb{Z}$ onto?

4. $f$ is one-to-one correspondence, or a bijection, if it is both one-to-one and onto.
   Examples:
   - Is $f(x) = x + 1$ from $\mathbb{Z}$ to $\mathbb{Z}$ bijective?
   - The identity function $\ell_A : A \rightarrow A$, $\ell_A(x) = x$, is a bijection?

5. If $f : A \rightarrow B$ is a bijection, then the function $f^{-1} : B \rightarrow A$ defined by $f^{-1}(b) = a$ if $f(a) = b$ is called the inverse function or $f$.
   Examples:
   - Let $f(x) = x + 1$, is $f$ invertible? If yes, what is $f^{-1}$?
   - Let $f(x) = x^2$, is $f$ invertible? If yes, what is $f^{-1}$?

\[\text{1The notation } \mathbb{Z} \text{ for the set of integers comes from the German word “Zahlen”}\]
6. Let $f_1$ and $f_2$ be two functions from $A$ to $\mathbb{R}$, then $f_1 + f_2$ and $f_1f_2$ are also functions from $A$ to $\mathbb{R}$ defined by

\[
(f_1 + f_2)(x) = f_1(x) + f_2(x),
\]
\[
(f_1f_2)(x) = f_1(x)f_2(x).
\]

Example: Let $f_1(x) = x^2$ and $f_2(x) = x - x^2$, then

\[
(f_1 + f_2)(x) = \quad (f_1f_2)(x) = \]

7. If $g : A \rightarrow B$ and $f : B \rightarrow C$, then define $h : A \rightarrow C$, $h(a) = f(g(a))$. $h$ is called the composite function of $f$ and $g$, and written as $f \circ g$.

Examples:

- Let $f(x) = 2x + 3$ and $g(x) = 3x + 2$,
  \[
  (f \circ g)(x) = f(g(x)) = 2(3x + 2) + 3 = 6x + 7 \\
  (g \circ f)(x) = g(f(x)) = 3(2x + 3) + 2 = 6x + 11
  \]
- Since $f^{-1}(b) = a$ when $f(a) = b$,
  \[
  (f^{-1} \circ f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a.
  \]

8. Two frequently used functions in computer science (translating real values to binary):

$f : \mathbb{R} \rightarrow Z$. Let $x \in \mathbb{R}$,

- floor function: $\lfloor x \rfloor =$ the largest integer $n$ such that $n \leq x$.
- ceiling function: $\lceil x \rceil =$ the smallest integer $n$ such that $n \geq x$.

Example: How many 8-bit packets can be transmitted in a second if the rate is 80 bits/sec? How many 8-bit packets can be transmitted in a second if the rate is 60 bits/sec?