1. A compound proposition is called a *tautology* if that is always true, no matter what the truth values of the propositions that occur in it.
   A compound proposition that is always false is called a *contradiction*.
   A proposition that is neither a tautology nor a contradiction is called a *contingency*.

2. The compound propositions p and q are called **logically equivalent** if $p \leftrightarrow q$ is a tautology.
   Notation: $p \equiv q$.

3. One way to determine whether two propositions are equivalent is to use a truth table.
   For example: De Morgans law:
   $$\neg(p \vee q) \equiv \neg p \wedge \neg q$$
   Verify by using truth table.

4. Some basic logical equivalences/laws, see Tables 6, 7, 8 in section 1.3 of Rosen (Propositional Equivalences).

Propositional functions – predicate and quantier

1. Propositional function $P(x)$: a statement involving the variables x
   Examples
   - Let $P(x)$ denote "$x > 3$", what are the truth values of $P(4)$ and $P(2)$?
   - Let $Q(x, y)$ denote "$x = y + 3$", what are the truth values of the proposition $Q(1, 2)$ and $Q(3, 0)$?

2. A statement of the form $P(x1, x2, ..., xn)$ is the value of the propositional function P at the n-tuple $(x1, x2, ..., xn)$, and P is called the *predicate*.

3. The *domain of discourse*, denoted U, is the set of values x that x is allowed to take in $P(x)$.

4. The *universal quantication* of $P(x)$ is the proposition
   "$P(x)$ is true for all value of x in U".
   Notation: $\forall x P(x)$, $\forall$ is called the universal quantier.
   Truth values of $\forall x P(x)$
   "$\forall x P(x)$" = True, when $P(x)$ is true for every x in U .
   "$\forall x P(x)$" = False, when there is an x in U for which $P(x)$ is false.

5. Examples:
   - "for all integers n, 2n is even" `(True)`
   - "for all real numbers x, $x^2 - 1 > 0$" `(False, e.g. x = 0)`
   - "for all CS major students S , S must take discrete math." `(True)`

6. The existential quantication of $P(x)$ is the proposition
   "There exists an element x in U such that $P(x)$ is true"
   Notation: $\exists x P(x)$. $\exists$ is called the *existential quantier*.
   Truth values of $\exists x P(x)$
   "$\exists x P(x)$" = True, when there is an x in U for which $P(x)$ is true.
   "$\exists x P(x)$" = False, when $P(x)$ is false for every x in U.

7. Examples
   - "there exists an integer $n, 2 * n$ is even" (True, in fact for all n)
   - "there exists a student S , S works hard" (True)
   - "there exists a real number $x, x^2 < 0$" (False)

8. Translating sentences into logical expressions. Examples: Let
   U = students in the class
   M (x) = "x has visited Mexico"
   C (x) = "x has visited Canada"
   Then
   - "Some student in this class has visited Mexico" $\exists x M(x)$.
   - "Every student in this class has visited either Canada or Mexico" $\forall x (C(x) \lor M(x))$.

9. The cornerstone arguments of logic:
   - $(p \land (p \to q)) \to q$ called *modus ponens* (the mode that affirms)
   - $(\neg q \land (p \to q)) \to \neg p$ called *modus tollens* (the mode that denies)