1. Assigning probabilities. A *probability distribution* for a set S $= \{s_1, s_2, ...s_n\}$ has two requirements:

   a) Pr(s_i) $\leq 1 \quad \forall s_i \in S$

   b) $\sum_{i=1}^{n}$ Pr(s_i) $= 1$.

2. The probability of an event (e.g. roll an even number on a die)
   Pr(E) $= \sum_{s_i \in E}$ Pr(s_i) $= |E|/|S|$

3. Recall Pr($\overline{E}$) $= 1-$ Pr(E).
   (Probability event does not happen is one minus probability it does happen.)

   Example: Probability your poker hand contains the Ace of Hearts. (Equals 1 minus the probability it does not contain it.)

4. **Combination of events.** Consider two events E_1 and E_2 in a sample space S. Then

   $$\text{Pr}(E_1 \bigcup E_2) = \text{Pr}(E_1) + \text{Pr}(E_2) - \text{Pr}(E_1 \bigcap E_2).$$

   Example: Probability a positive integer not exceeding 100 is even and divisible by 5.

5. **Conditional Probability.** Let E and F be events, then the probability of event E given that event F has occurred (probability of E conditioned on F):

   $$\text{Pr}(E \mid F) = \frac{Pr(E \bigcap F)}{Pr(F)}.$$

   Note, conditional probability, Pr($E \mid F$), is looking at a *sequence of events*: given that event F has occurred, what is the probability event E will occur? In contrast, combination of two events Pr($E_1 \bigcup E_2$) asks what is the probability that *both events hold simultaneously*?

6. **Independent events**. If E and F are independent:

   $$\text{Pr}(E \bigcap F) = \text{Pr}(E) \cdot \text{Pr}(F), \quad \text{and}$$

   $$\text{Pr}(E \mid F) = \frac{Pr(E) \cdot Pr(F)}{Pr(F)} = Pr(E).$$

   Example: Tossing a fair coin. Does the next outcome depend on the past outcomes?

7. *The Monty Hall Problem*. (The basis of the game show “Let’s make a Deal”.) Is the *optimal strategy* to not switch or to switch? What happens when there are more than three doors?