1. The **product rule**: Suppose that a procedure can be broken into a sequence of two tasks. If there are n_1 ways to do the first task, and for each of the ways of doing the first task, there are n_2 ways to do the second task, then there are $n_1 n_2$ ways to do the procedure.
   In set notation: Let A_1 be the set of the n_1 ways to do the first task, and A_2 the set of n_2 ways to do the second task. Then
   $$|A_1 \times A_2| = |A_1| \times |A_2| = n_1 n_2.$$

2. The **extended product rule**: $|A_1 \times A_2 \times \cdots \times A_m| = |A_1| \times |A_2| \times \cdots \times |A_m|$.
   Examples:
   (a) How many numbers in the range $1000 - 9999$?
   (b) How many numbers in the range $1000 - 9999$ do not have any repeated digits?
   (c) How many one-to-one functions are there from a set with m elements to a set with n elements?
   (d) Recall cardinality of power set of a set A: $|P(A)| = 2^{|A|}$ (the number of subsets).
   (e) How many 10-digit telephone numbers of the form NXX-NXX-$XXXX$, where $N \in \{2, 3, \ldots, 9\}$ and $X \in \{0, 1, \ldots, 9\}$?

3. The **sum rule**: If a task can be done either in one of n_1 ways or in one of n_2 ways, where none of the set of n_1 ways are the same as any of the n_2 ways, then there are $n_1 + n_2$ ways to do the task.
   In set notation: Let A_1 be the set of the first n_1 ways to do a task, and A_2 the set of the second n_2 ways to do the task. Assume that A_1 and A_2 are disjoint, i.e., $A_1 \cap A_2 = \emptyset$, then the set of ways to do the task is $A_1 \cup A_2$ and the number of ways is
   $$|A_1 \cup A_2| = |A_1| + |A_2| = n_1 + n_2.$$

4. The **extended sum rule**: $|A_1 \cup A_2 \cup \cdots \cup A_m| = |A_1| + |A_2| + \cdots + |A_m|$ provided that $A_i \cap A_j = \emptyset$ whenever $i \neq j$.

   Example: How many ways can we choose one student from the CS dept if there are f freshman, p sophomores, j juniors and s seniors?

5. The **inclusion-exclusion rule**: Suppose that a task can be done in n_1 or n_2 ways, but that some of the set of n_1 ways to do the task are the same as some of the n_2 ways. First add n_1 and n_2 ways, then subtract the number of common ways.
   In set notation:
   $$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|.$$
   Example: How many bit strings of length eight start with a 1 bit or end with two bits 00?

6. **More complex counting problems**: many counting problems cannot be solved using just the sum rule or the product rule, but can be solved using both of these rules in combination.
   Examples:
   (a) Let P_6 denote the number of passwords of six characters, where each character is an uppercase letter or digit, and each password must contain at least one digit. What's P_6?
   (b) How many such passwords are there of length six to eight characters long?
   (c) Number of IPv4 Internet addresses.

7. Tying it all together – e.g., problem 5.1.21.

8. **The Pigeonhole Principle**: If $k + 1$ or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.
   Example: Among any group of 367 people, there must be at least two people with the same birthday, because there are only 366 possible birthdays.