1) a) The inductive hypothesis \( P(n) \) is that \( 1^3 + 2^3 + \ldots + n^3 = (n(n+1)/2)^2 \) for any \( n \in \mathbb{Z}^+ \).

b) Plug the number 1 into both sides and find \( 1^3 = ((1 \cdot 2)/2)^2 \). In other words, both sides of the expression evaluate to the number 1.

c) We want to show for any \( n \geq 1 \) that \( P(n) \rightarrow P(n+1) \). In other words, write the raw expression for \( P(n+1) \) in terms of the raw expression for \( P(n) \) and use the inductive hypothesis:

\[
1^3 + 2^3 + \ldots + (n+1)^3 = \left(1^3 + 2^3 + \ldots + (n)^3\right) + (n+1)^3
= \left(n(n+1)/2\right)^2 + (n+1)^3 \quad \text{i.e. Assume } P(n)
= (n+1)^2 \left(\frac{n^2}{4} + n + 1\right) = (n+1)^2 \left(\frac{n^2 + 4n + 4}{4}\right)
= \left(\frac{(n+1)(n+2)}{2}\right)^2.
\]

2) a) By computing the first few sums we get the answers 1/2, 2/3, and 3/4. So now we have a hypothesis, namely that

\[
\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}
\]

b) Proof by induction has three steps: (1) identify the hypothesis (part a above); (2) Prove the base case; (3) Prove that \( P(n) \rightarrow P(n+1) \). We did step (1) in part a.

- Prove the base case: If \( n = 1 \) then \( 1/2 = 1/2 \). So the bases step holds.

- Prove the inductive step:

\[
\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} = \frac{n}{n+1} + \frac{1}{(n+1)(n+2)}
= \frac{n(n+2) + 1}{(n+1)(n+2)} = \frac{(n+1)^2}{(n+1)(n+2)} = \frac{(n+1)}{(n+2)}
\]

Therefore, \( \forall n \in \mathbb{Z}^+, P(n) \).
3) Proof by induction has three steps: (1) identify the hypothesis; (2) Prove the base case; (3) Prove the inductive step that \( P(n) \rightarrow P(n + 1) \).

- Inductive hypothesis: \( \sum_{j=0}^{n} ar^j = \frac{ar^{n+1} - a}{(r-1)} \)
- Base case: Check that \( P(0) \) is true: \( ar^0 = \frac{(ar-a)}{(r-1)} = \frac{a(r-1)}{(r-1)} \). Simplifying both sides yields \( a = a \).
- Inductive step:

\[
\sum_{j=0}^{n+1} ar^j = \sum_{j=0}^{n} ar^j + ar^{n+1} \\
= \frac{ar^{n+1} - a}{(r-1)} + ar^{n+1} = \frac{ar^{n+1} - a + ar^{n+1}(r-1)}{(r-1)} \\
= \frac{ar^{n+1} - a + ar^{n+2} - ar^{n+1}}{(r-1)} = \frac{ar^{n+2} - a}{(r-1)}
\]

Therefore, \( \forall n \in \mathbb{N} P(n) \).

4)

The basis is for \( n = 2 \), which is true by the distributive law (can be demonstrated using, say, a membership table). We assume that \( P(n) : ((A_1 \cap A_2 \cap \ldots \cap A_n) \cup B) = (A_1 \cup B) \cap (A_2 \cup B) \cap \ldots \cap (A_n \cup B) \) is true. Then, for \( P(n + 1) \) we have

\[
(A_1 \cap A_2 \cap \ldots \cap A_n \cap A_{n+1}) \cup B = ((A_1 \cap A_2 \cap \ldots \cap A_n) \cap A_{n+1}) \cup B \\
= ((A_1 \cap A_2 \cap \ldots \cap A_n) \cup (A_{n+1} \cup B)) \\
= (A_1 \cup B) \cap (A_2 \cup B) \cap \ldots \cap (A_n \cup B) \cap (A_{n+1} \cup B)
\]

Here in bold letters we indicate the use of the inductive assumption.
5) 

a) 
\[ f(2) = f(1) - f(0) = 1 - 1 = 0, \]
\[ f(3) = f(2) - f(1) = 0 - 1 = -1, \]
\[ f(4) = f(3) - f(2) = -1 - 0 = -1, \]
\[ f(5) = f(4) - f(3) = -1 - 1 = 0. \]

b) 
Clearly \( f(n) = 1 \) for all \( n \), since \( 1 \cdot 1 = 1 \).

c) 
\[ f(2) = f(1)^2 + f(0)^3 = 1^2 + 1^3 = 2, \]
\[ f(3) = f(2)^2 + f(1)^3 = 2^2 + 1^3 = 5, \]
\[ f(4) = f(3)^2 + f(2)^3 = 5^2 + 2^3 = 33, \]
\[ f(5) = f(4)^2 + f(3)^3 = 33^2 + 5^3 = 1214. \]

d) 
Clearly \( f(n) = 1 \) for all \( n \), since \( 1/1 = 1 \).

6) 
- The inductive hypothesis \( P(n) \) is that \( f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1} \).
- The base case: \( f_1^2 = f_1 f_2 \). Since \( f_1 = f_2 = 1 \) we get \( 1 = 1 \).
- The inductive step:

\[
\begin{align*}
  f_1^2 + f_2^2 + \cdots + f_{n+1}^2 &= f_1^2 + f_2^2 + \cdots + f_n^2 + f_{n+1}^2 \\
  &= f_n f_{n+1} + f_{n+1}^2 = f_{n+1} (f_n + f_{n+1}) = f_{n+1} f_{n+2}.
\end{align*}
\]

Therefore, \( \forall n \in \mathbb{Z}^+ \), \( P(n) \).
7) For the basis step, \( n = 1 \), \( f_2 f_0 - f_1^2 = 1 \times 0 - 1^2 = -1 = (-1)^1 \). The inductive step is then:

\[
f_{n+2}f_n - f_{n+1}^2 = (f_{n+1} + f_n)f_n - f_{n+1}^2 \\
= f_{n+1}f_n + f_n^2 - f_{n+1}^2 \\
= -f_{n+1}(f_{n+1} - f_n) + f_n^2 \\
= -f_{n+1}f_{n-1} + f_n^2 \\
= -(f_{n+1}f_{n-1} - f_n^2) \\
= -(-1)^n \\
= (-1)^{n+1}
\]

8) procedure recurseSum(n: positive integer)
   if (n=1) then recurseSum(n):=1;
   else recurseSum(n) := n + recurseSum(n-1);
   end

9) Prove correctness of the algorithm in problem 8. Use proof by induction:
   • Basis step: recurseSum(1) = 1.
   • Inductive Hypothesis: recurseSum(n) is correct.
   • Show recurseSum(n) \( \rightarrow \) recurseSum(n+1). Given the argument (n+1) the function executes the else statement and (n+1) is added to recurseSum(n) which, by inductive hypothesis, we assumed correctly calculated the sum of the first n positive integers. Hence recurseSum(n+1) adds (n+1) to the sum of the first n positive integers, thus correctly finding the sum of the first (n+1) positive integers.
10) 

procedure recurseSeq(n: non-negative integer) 
    if n=0 
        return 1 
    elseif n=1 
        return 2 
    else 
        return recurseSeq(n-1)*recurseSeq(n-2) 
    endif 

11) 

procedure iterateSeq(n: non-negative integer) 
    if n=0 
        y=1 
    else 
        x=1 
        y=2 
        for i = 1 to (n-1) 
            z = x·y 
            x = y 
            y = z 
        endfor 
    endif 
    return y