1) What rule of inference is used in each of these arguments?

a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

2) Use rules of inference to show that the hypotheses “If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on,” “If the sailing race is held, then the trophy will be awarded,” and “The trophy was not awarded” imply the conclusion “It rained.” [Hint: Build an argument like the in-class example]

3) For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

a) “If I play hockey, then I am sore the next day.” “I use the whirlpool if I am sore.” ”I did not use the whirlpool.”

b) “If I work, it is either sunny or partly sunny.” “I worked last Monday or I worked last Friday.” “It was not sunny on Tuesday.” “It was not partly sunny on Friday.”

c) “All insects have six legs.” “Dragonflies are insects.” “Spiders do not have six legs.” “Spiders eat dragonflies.”

d) “Every student has an Internet account.” “Homer does not have an Internet account.” “Maggie has an Internet account.”

e) “All foods that are healthy to eat do not taste good.” “Tofu is healthy to eat.” “You only eat what tastes good.” “You do not eat tofu.” “Cheeseburgers are not healthy to eat.”
f) “I am either dreaming or hallucinating.” “I am not dreaming.” “If I am hallucinating, I see elephants running down the road.”

4) Show that the equivalence \( p \land \neg p \equiv F \) can be derived using resolution together with the fact that a conditional statement with a false hypothesis is true. [Hint: Let \( q = r = F \) in resolution. Note “conditional statement” is another term for “implication”]

5) The Logic Problem, taken from WFF’N PROOF, The Game of Logic, has these two assumptions:

1. “Logic is difficult or not many students like logic.”
2. “If mathematics is easy, then logic is not difficult.”

By translating these assumptions into statements involving propositional variables and logical connectives, determine whether each of the following are valid conclusions of these assumptions:

a) That mathematics is not easy, if many students like logic.

b) That not many students like logic, if mathematics is not easy.

6) Show that the additive inverse, or negative, of an even number is an even number using a direct proof.

7) Prove that if \( n \) is an integer and \( 3n + 2 \) is even, then \( n \) is even using

a) a proof by contraposition.

b) a proof by contradiction.

8) Prove the proposition \( P(1) \), where \( P(n) \) is the proposition “If \( n \) is a positive integer, then \( n^2 \geq n \).” What kind of proof did you use? [Hint: This is very simple.]

9) Show that at least 3 of any 25 days chosen must fall in the same month of the year.

10) Use a proof by cases to show that \( \min(a, \min(b, c)) = \min(\min(a, b), c) \) whenever \( a, b, \) and \( c \) are real numbers.

11) Prove that there is a positive integer that equals the sum of the positive integers not exceeding it. Is your proof constructive or nonconstructive?

12) Prove that there are no solutions in integers \( x \) and \( y \) to the equation \( 2x^2 + 5y^2 = 14 \).