1) Which of these are propositions? What are the truth values of those that are propositions?
   a) What time is it?
   b) I am telling a lie.
   c) There is no sand at the beach.
   d) \(4 + x = 5\).
   e) Dogs can fly.
   f) \(2^n \geq 100\).

2) Let \(p\), \(q\), and \(r\) be the propositions
   \(p\) : You have the flu.
   \(q\) : You miss the final examination.
   \(r\) : You pass the course.

Express each of these propositions as an English sentence.
   a) \(p \rightarrow q\)
   b) \(\neg q \leftrightarrow r\)
   c) \(q \rightarrow \neg r\)
   d) \(p \lor q \lor r\)
   e) \((p \rightarrow \neg r) \lor (q \rightarrow \neg r)\)
   f) \((p \land q) \lor (\neg q \land r)\)

3) Let \(p\), \(q\), and \(r\) be the propositions
   \(p\) : You get an A on the final exam.
   \(q\) : You do every exercise in this book.
   \(r\) : You get an A in this class.

Write these propositions using \(p\), \(q\), and \(r\) and Boolean logical connectives.
   a) You get an A in this class, but you do not do every exercise in this book.
   b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
c) To get an A in this class, it is necessary for you to get an A on the final.

d) You get an A on the final, but you don’t do every exercise in this book; nevertheless, you get an A in this class.

e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

4) Construct a truth table for each of these compound propositions.
   a) \((p \lor q) \lor r\)
   b) \((p \lor q) \land \neg r\)

5) Use truth tables to verify the associative laws
   a) \((p \lor q) \lor r \equiv p \lor (q \lor r)\)
   b) \((p \land q) \land r \equiv p \land (q \land r)\)

6) Show that \([\neg p \land (p \lor q)] \to q\) is a tautology by using a truth table.

7) Show that \(\neg p \to (q \to r)\) and \(q \to (p \lor r)\) are logically equivalent by using truth tables.

8) Let \(P(x)\) be the statement “the word \(x\) contains the letter a.” What are the truth values?
   a) \(P(\text{orange})\)
   b) \(P(\text{lemon})\)
   c) \(P(\text{true})\)
   d) \(P(\text{false})\)

9) Consider each of the values of the variable \(x\) listed below and the predicate \(P(x)\), where \(P(x)\) is the statement “\(x > 1\)”. State the value of \(x\) after the statement if \(P(x)\) then \(x := 1\) is executed. (Recall from Lec 1 slides, page 22, the operator “:=” means “is defined as” or “is allocated the value”.)
   a) \(x = 0\).
   b) \(x = 1\).
   c) \(x = 2\).
10) Let $N(x)$ be the statement “$x$ has visited North Dakota,” where the domain consists of the students in your school. Express each of these quantifications in English.

a) $\exists x N(x)$.

b) $\forall x N(x)$.

c) $\neg \exists x N(x)$.

d) $\exists x \neg N(x)$.

e) $\neg \forall x N(x)$.

f) $\forall x \neg N(x)$.

11) Let $Q(x)$ be the statement “$x + 1 > 2x$.” If the domain consists of all integers, what are these truth values?

a) $Q(0)$.

b) $Q(-1)$.

c) $Q(1)$.

d) $\exists x Q(x)$.

e) $\forall x Q(x)$.

f) $\exists x \neg Q(x)$.

g) $\forall x \neg Q(x)$. 