Mayra Vega
MAE 216- Statistical Thermodynamics
June 18, 2012

Biasing Brownian motion from thermal ratchets

Introduction

Brownian motion is the random movement of small particles however, by understanding the thermodynamics of this motion it is possible to bias the motion of these particles in a desired direction (Astumian R. , 1997). Biasing the motion involves the use of thermal ratchets. These thermal ratchets take advantage of fluctuations in potentials to induce movement of particles. These potentials can arise from noise, chemical reactions, or an externally applied voltage. What makes these ratchets remarkable is that movement of particles can be accomplished even if an externally applied force averages to zero. (Astumian R. , 1997). Chemical processes for example can induce flow of particles as long as the fluctuation of the chemical force is not too short compared to the relaxation time (Bier, 1994). The energy from these fluctuations can also be used to perform work. One application of using this process is to separate particles. (Astumian M. B., 1996).This paper will briefly discuss the basics of Brownian motion, how biasing the Brownian motion can be used to separate particles and discuss future work.

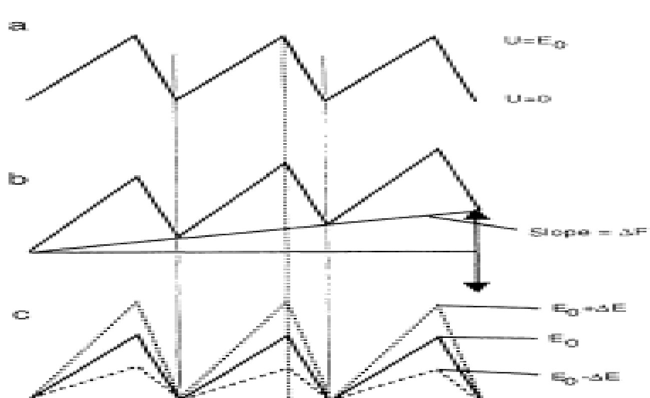
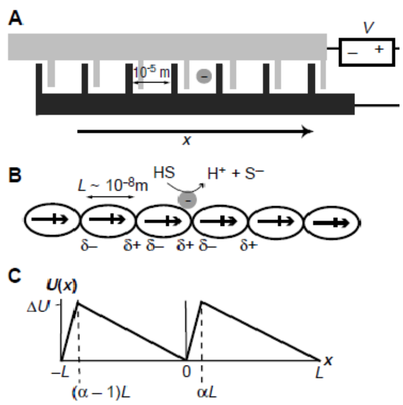


Fig 1. Feynman's ratchet. (a) Demonstrates the piecewise Linear potential. (b) Fluctuation of the force. (c) Fluctuation of the amplitude.

What are Thermal Ratchets?

To understand thermal ratchets we look at the characteristics of the Feynman Ratchet. The ratchet describe a piecewise linear potential and demonstrates that the potential fluctuates. However what allows the movement of particles is an asymmetric potential and the fluctuation of not just the potential, but the amplitude of the potential or the 'teeth' of the ratchet. (Bier, 1994) Manipulating the ratchet's teeth will make the motion of particles more likely in one direction rather than the other. (Astumian M. B., 1996) There are multiple ways to achieve an anisotropic potential and a few are described below:



1. Applying a voltage across an interdigitated electrode with anisotropically positioned barriers on a glass side.

2. As mentioned, brownian motion can be achieved without an external force. Another method for inducing this motion is with an array of dipoles on which a charged Brownian particle moves, catalyze a reaction and induce a fluctuating electrostatic potential energy . (Astumian R. , 1997)

Fig 2. Ways to achieve an anisotropic potential. (A) corresponds to the process described by 1. (B) corresponds to the process described by 2. (C) demonstrates the potential in both A and B where ΔU is the potential. As mentioned, it is the fluctuation of the amplitude that causes Brownian motion. This fluctuation can be achieved by using an external switching device to control the applied voltage. (Astumian R. , 1997)

Biasing Brownian Motion

The following equation describes brownian motion:

$$\beta \frac{dx}{dt} = -\frac{\partial}{\partial x}[g(t)U(x)] + f\,(t) + \left(\beta\sqrt{2D}\right)\xi(t) \text{ (Astumian M. B., 1996)}$$

Where β describes the coefficient of friction, $\xi(t)$ describes the zero averaged noise, and D controls the amplitude of the noise. B and D are related by the fluctuation-dissipation theorem where $D = kt/\beta$. The functions f(t) describes the additive noise and g(t) describe the multiplicative noise. Movement can be achieved when the multiplicative noise varies in time. Therefore g(t) will be a fluctuating function but f(t) is zero meaning that no external force is applied. (Astumian M. B., 1996) To visualize what is happening, recall figure 1. When an asymetric voltage is applied and varies between 'on' and 'off' cycles for the same period of time, motion of particles is induced ‘downhill’. (juliette Rousselet, 1994) This results in the molecules moving in a single direction. Thus, by flipping the potential between positive $V_+\ (x)$and negative $V_-(x)$, flow reversal can be achieved (Astumian M. B., 1996). The literature uses the Fokker-Planck equation, probability theory, re-dimensionalization and scaling to develop the formulas for the flux describing the movement of molecules for high frequency and low frequency domains are derived. The formulas that arise from this are as follows:

$$J_{min} = \frac{\lambda}{(2\lambda+1)(\lambda+1)}\frac{EkT}{\beta L}$$ (Astumian M. B., 1996)

$$J_{max} = \frac{2}{(2\lambda+1)}\frac{kT}{\beta L}$$ (Astumian M. B., 1996)

Where λ describes the time spent in V_o (which is the flat potential between) V_+ (x) and V_- (x), or the flipping rate, E is the energy difference between the minimum (wells) and the maximum (peaks), k is the Boltzmann constant, T is temperature, and L is length of a period. $-3 < \log \gamma < 0$ is the low frequency range and $0 < \log \gamma < 3$ is the high frequency range where γ is the the speed of the noise. E is usually taken to be 10 but it is evident that the coefficient of friction will influence the flux of particles. (Astumian M. B., 1996). This is what induces separation of particles as particles with smaller values for β will move faster than those with larger values. (Astumian M. B., 1996). The characteristics and sizes of the molecules will influence the path of these molecules, but is also evident that a flipping rate is required for motion to occur.

Future Work and Conclusions

The implications of the literature suggests motion of particles can be induced without the need of an external force. By harnessing energy from chemical reactions we can use it do work (Bier, 1994). Also, instead of reducing noise, it can be utilized to generate Brownian motion (Astumian R. , 1997). Future work is still required to understand the engineering aspects involved with designing these molecular pumps which are used to separate particles. Once we are able to design these and optimize their performance they could have strong implications for bio-chemical processes and in electromagnetic communications.

Bibliography

Astumian, M. B. (1996). Biasing Brownian Motion in Different Directions in a 3-state fluctuating potential and an application for the separation of small particles . *Physics Review Letters* , 4277-4280.

Astumian, R. (1997). Thermodynamics and Kinetics of a Brownian Motor. *Science* , 917-922.

Bier, R. D. (1994). Fluctuation Driven Ratchets: Molecular Motors . *Physics Review Letters* , 1766-1769.

juliette Rousselet, L. S. (1994). Directional motion of brownian particles induce by a periodic asymmetric potential. *Nature* , 446-448.