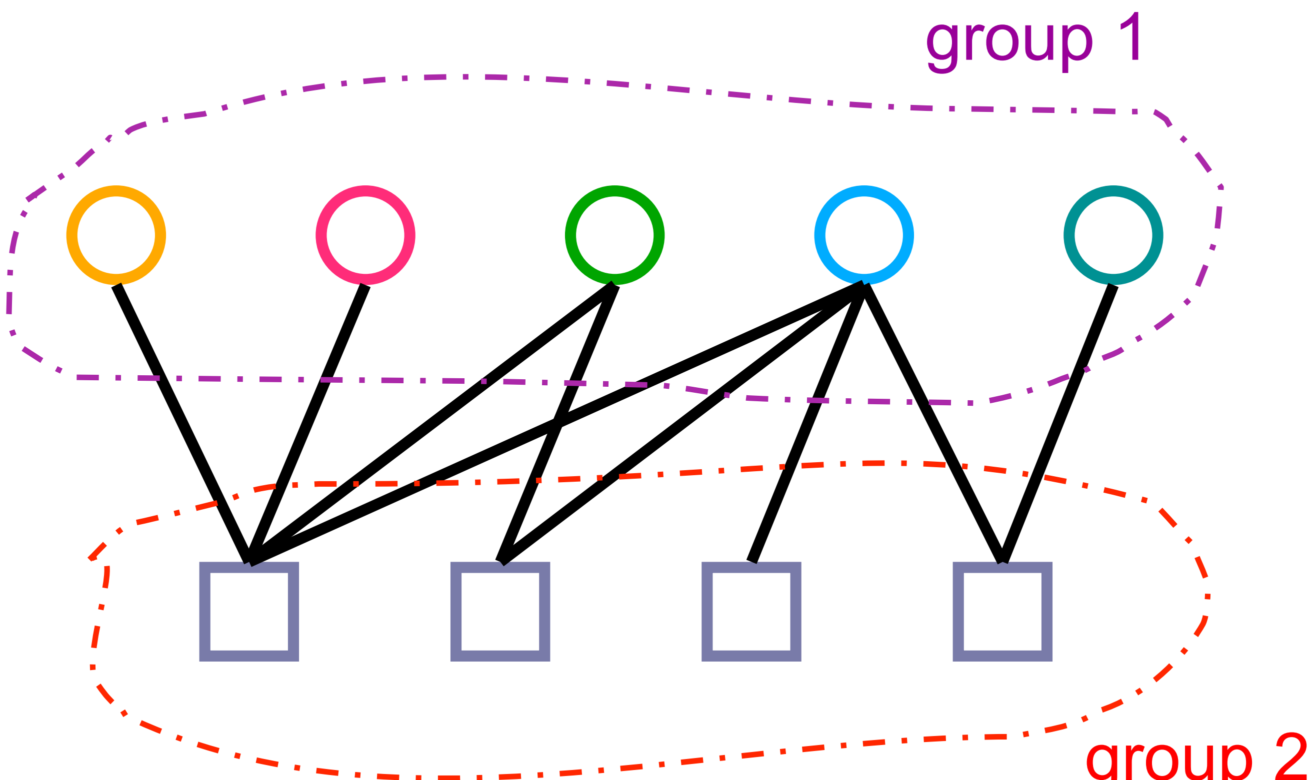
ECS 253 / MAE 253, Lecture 11

May 3, 2016



"Bipartite networks, trees, and cliques" &
"Flows on spatial networks"

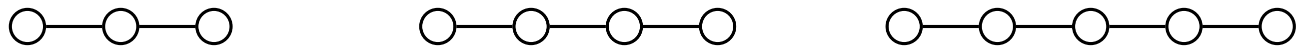
Other important basic networks

- Bipartite networks
- Hypergraphs
- Trees
- Planar graphs
- Cliques

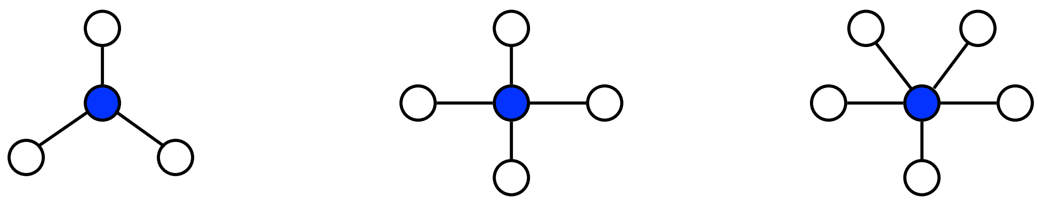
This content largely from Adamic's lectures

Some Basic Types of Graphs

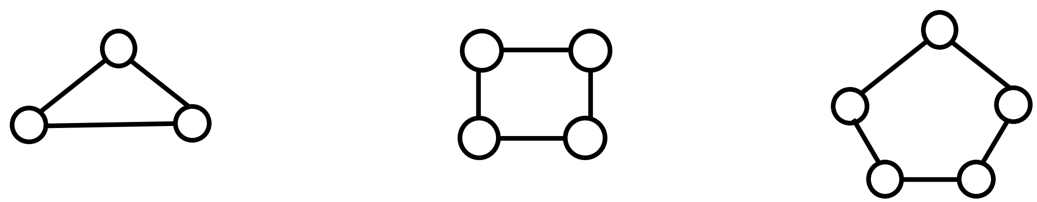
Paths



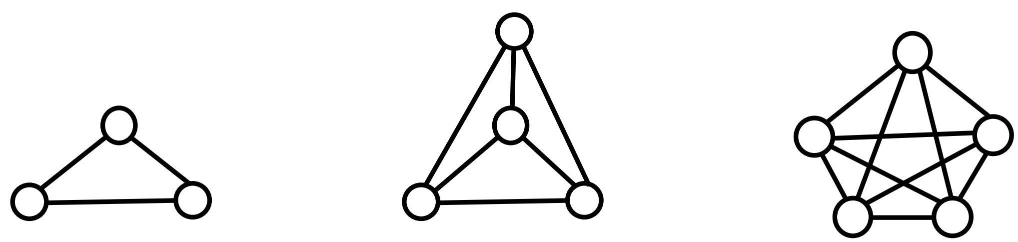
Stars



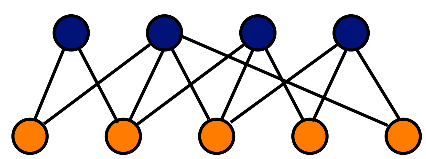
Cycles



Complete Graphs



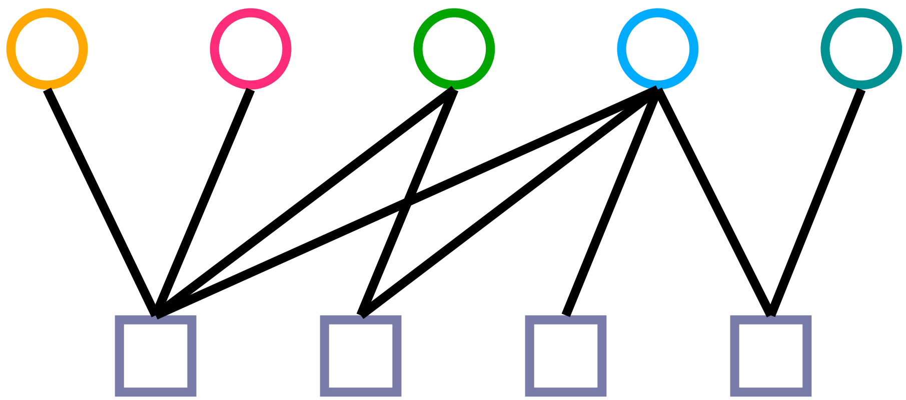
Bipartite Graphs



Bipartite (two-mode) networks

- edges occur only between two groups of nodes, not within those groups

- for example, we may have individuals and *events*
  - directors and boards of directors
  - customers and the items they purchase
  - metabolites and the reactions they participate in



Fun websites for bipartite graphs

- The Oracle of Bacon – The path to Kevin Bacon
  http://oracleofbacon.org/

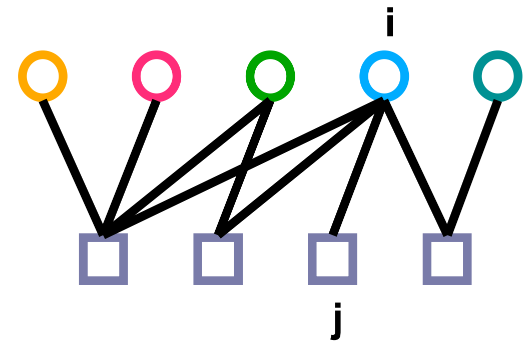
- SixDegrees.org – connecting causes and celebrities
  http://www.sixdegrees.org/

- Six degrees of Kevin Garnett
  http://www.slate.com/articles/sports/slate_labs/2013/10/six_degrees_of_kevin_garnett_connect_any_two_athletes_who_ve_ever_played.html

- Six degrees of NBA seperation
  http://harvardsportsanalysis.wordpress.com/2011/03/04/six-degrees-of-nba-separation/
  (Blog post explaining use of Dijkstra's algorithm)

in matrix notation

- B_{ij}
  - = 1 if node i from the first group
    links to node j from the second group
  - = 0 otherwise

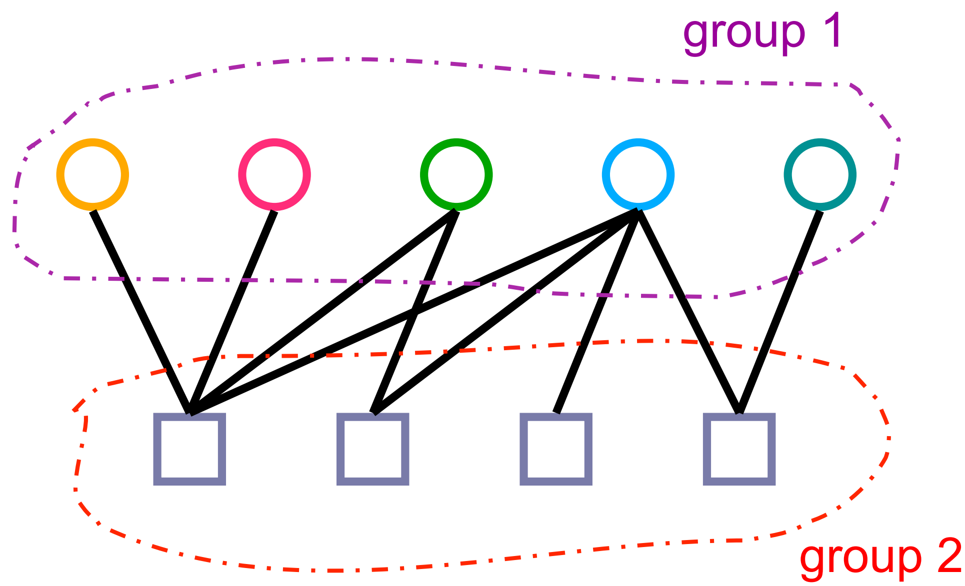


- B is usually not a square matrix!
  - for example: we have n customers and m products

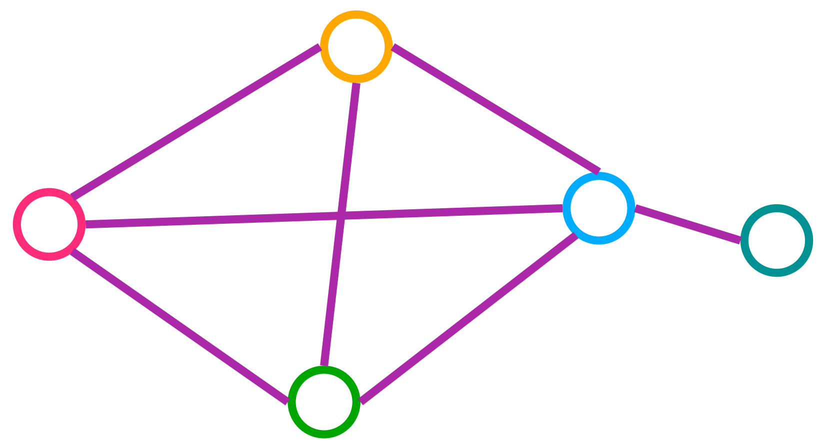
$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

going from a bipartite to a one-mode graph

- Two-mode network

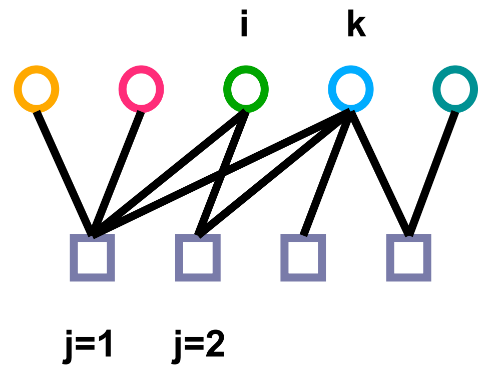


- One mode projection
  - two nodes from the first group are connected if they link to the same node in the second group
  - naturally high occurrence of cliques
  - some loss of information
  - Can use weighted edges to preserve group occurrences



Collapsing to a one-mode network

- i and k are linked if they both link to j
- $P_{ij} = \sum_k B_{ki} B_{kj}$
- $P' = B B^T$
  - the transpose of a matrix swaps B_{xy} and B_{yx}
  - if B is an *nxm* matrix, B^T is an *mxn* matrix



$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

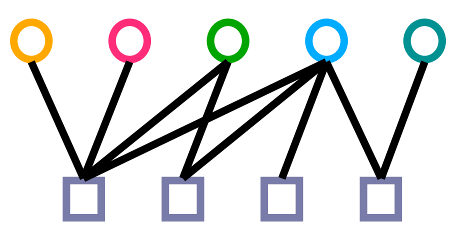
$$B^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

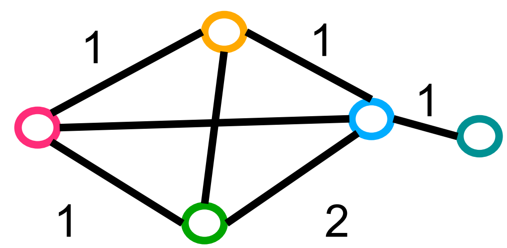
Matrix multiplication

- general formula for matrix multiplication $Z_{ij} = \sum_k X_{ik} Y_{kj}$
- let $Z = P'$, $X = B$, $Y = B^T$

$$P' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 2 & 0 \\ 1 & 1 & 2 & 4 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = 1*1+1*1 + 1*0 + 1*0 = 2$$



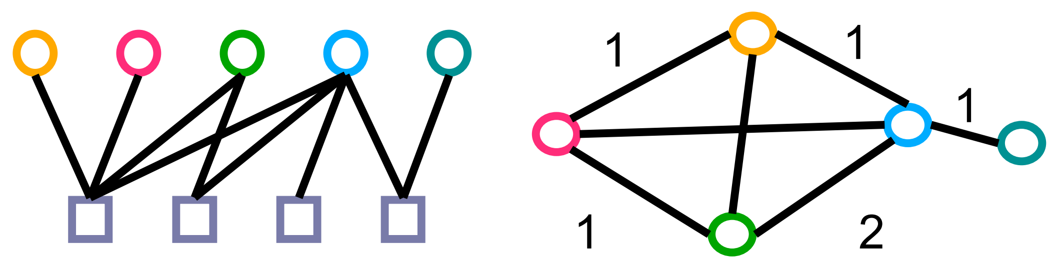


Collapsing a two-mode network to a one mode-network

- Assume the nodes in group 1 are people and the nodes in group 2 are movies
- P' is symmetric
- The diagonal entries of P' give the number of movies each person has seen
- The off-diagonal elements of P' give the number of movies that both people have seen

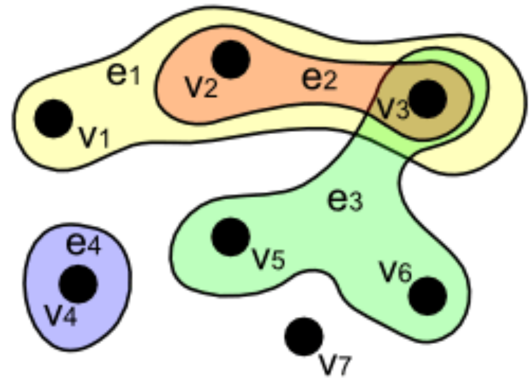
P' =

1	1	1	1	0
1	1	1	1	0
1	1	2	2	0
1	1	2	4	1
0	0	0	1	1

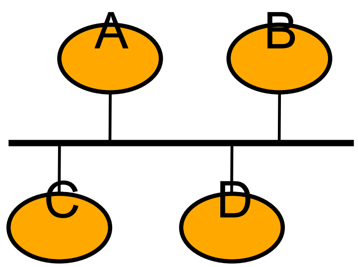


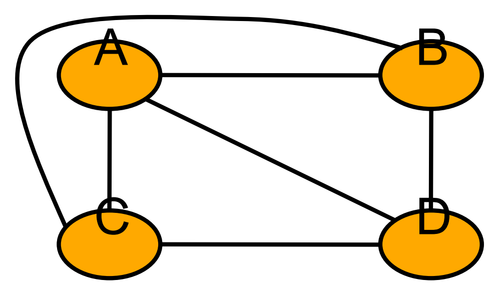
HyperGraphs

- Edges join more than two nodes at a time (*hyperEdge*)
- Affliation networks



- Examples
  - Families
  - Subnetworks





Hypergraphs — beyond dyadic iteractions

In ways, good models of social networks.

- “Complex Networks as Hypergraphs”
  Ernesto Estrada, Juan A. Rodriguez-Velazquez
  arXiv:physics/0505137, 2005.

- “Random hypergraphs and their applications”,
  G Ghoshal, V Zlatić, G Caldarelli, MEJ Newman,
  Physical Review E 79 (6), 2009.

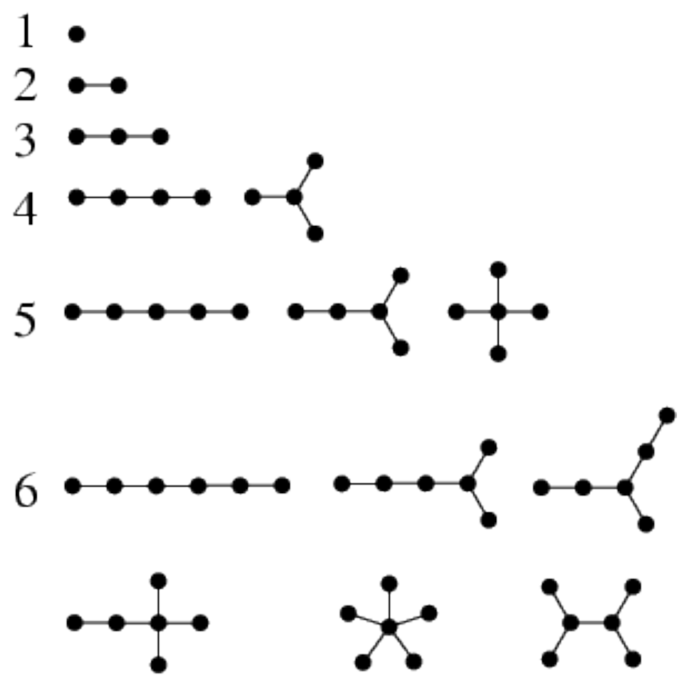
- Ramanathan, R., et al. “Beyond graphs: Capturing groups in networks.”
  NetSciCom, 2011 IEEE Conference on. IEEE, 2011.

- “Information Flows: A Critique of Transfer Entropies”
  Ryan G. James, Nix Barnett, James P. Crutchfield
  Accepted to Physical Review Letters, April 12, 2016.

- See also hypergraph mining, hypergraph learning algorithms, overlapping communities, link prediction...

Trees

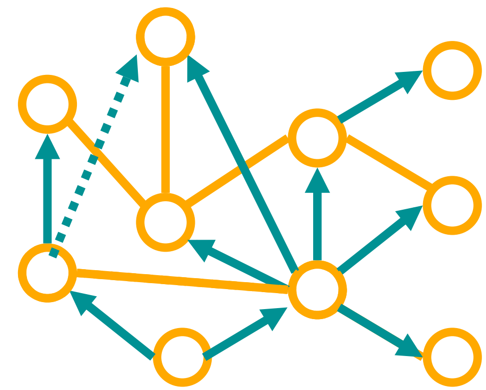
- Trees are undirected graphs that contain no cycles



- For n nodes, number of edges m = n-1
- Any node can be dedicated as the root

examples of trees

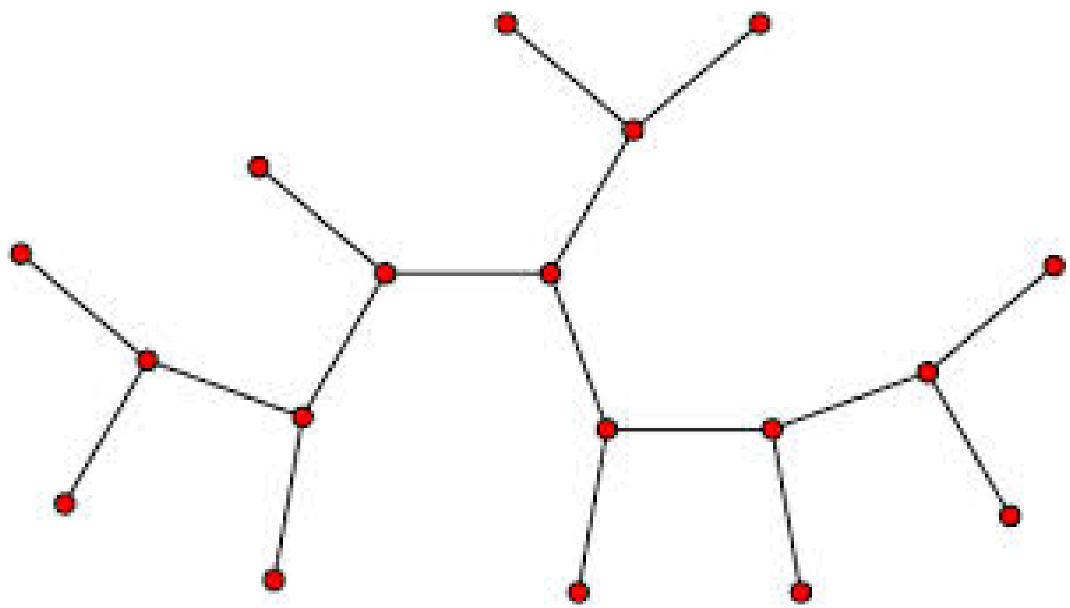
- In nature
  - trees
  - river networks
  - arteries (or veins, but not both)
- Man made
  - sewer system
- Computer science
  - binary search trees
  - decision trees (AI)
- Network analysis
  - minimum spanning trees
    - from one node – how to reach all other nodes most quickly
    - may not be unique, because shortest paths are not always unique
    - depends on weight of edges



Searching on a tree

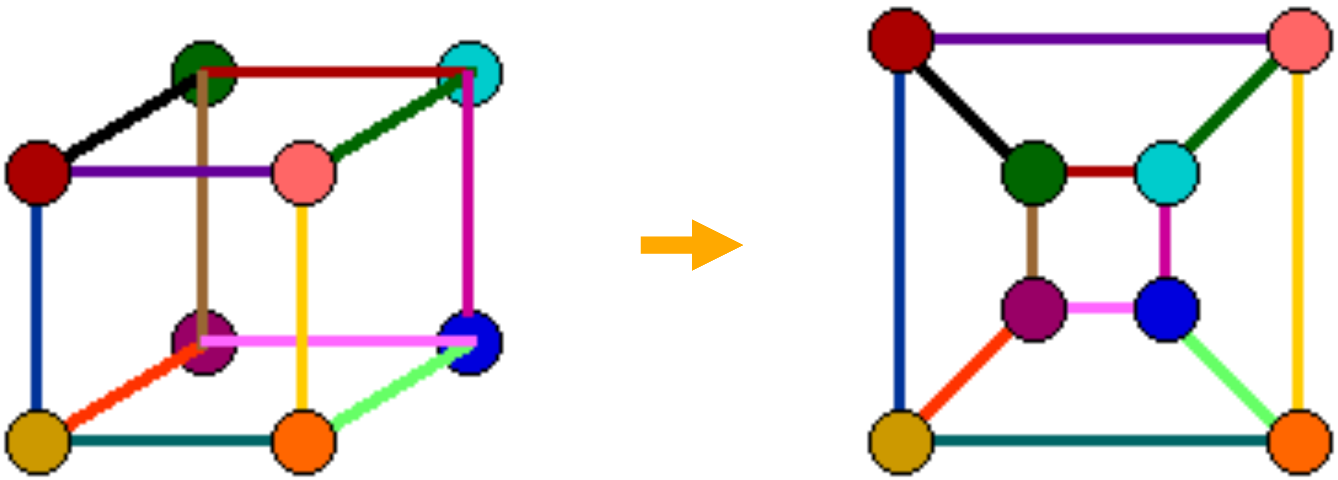
- Breadth first search:
  explore all the neighbors first

- Depth first search:
  take a step out in hop-count each iteration

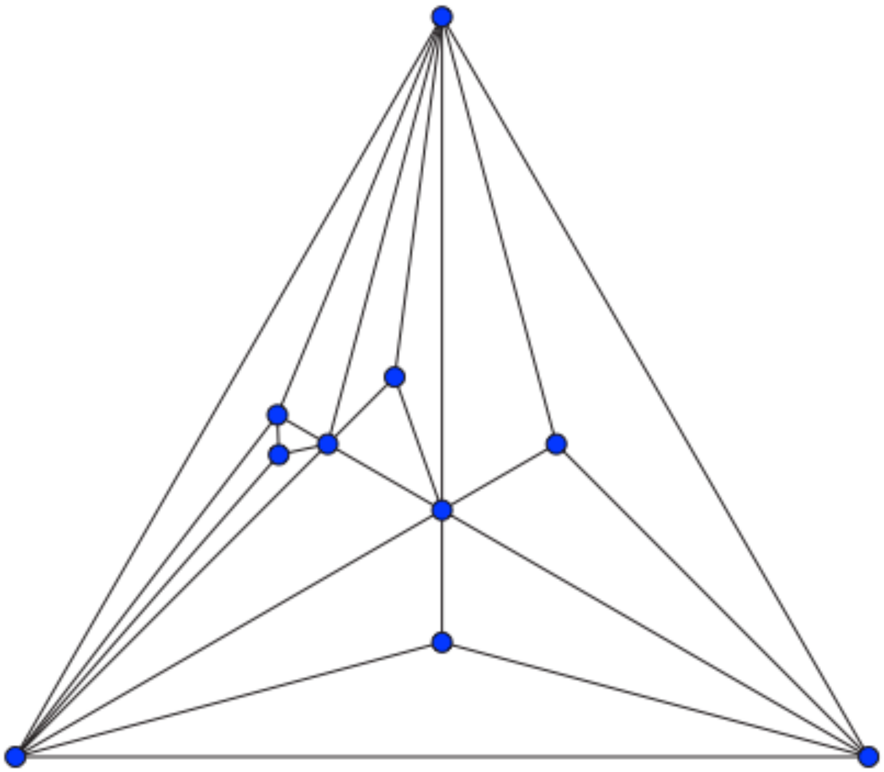


Planar graphs

- A graph is planar if it can be drawn on a plane without any edges crossing



Apollonian network



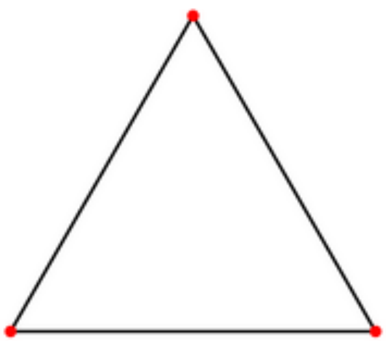
- An undirected graph formed by a process of recursively subdividing a randomly selected triangle into three smaller triangles.

- A planar graph with power law degree distribution, and small world property.

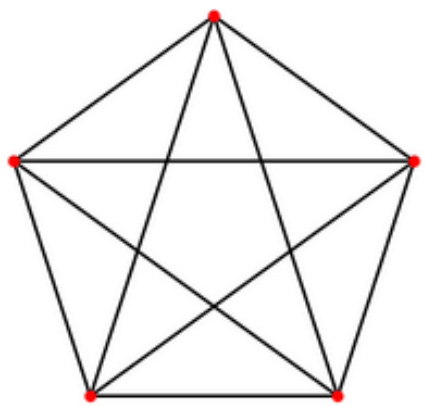
- A planar 3-regular graph, and uniquely 4-colorable.

Cliques and complete graphs

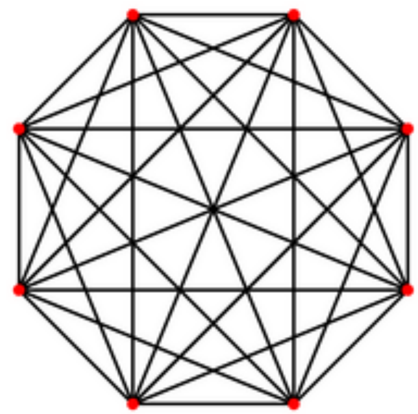
- K_n is the complete graph (clique) with K vertices
  - each vertex is connected to every other vertex
  - there are n*(n-1)/2 undirected edges



K_3

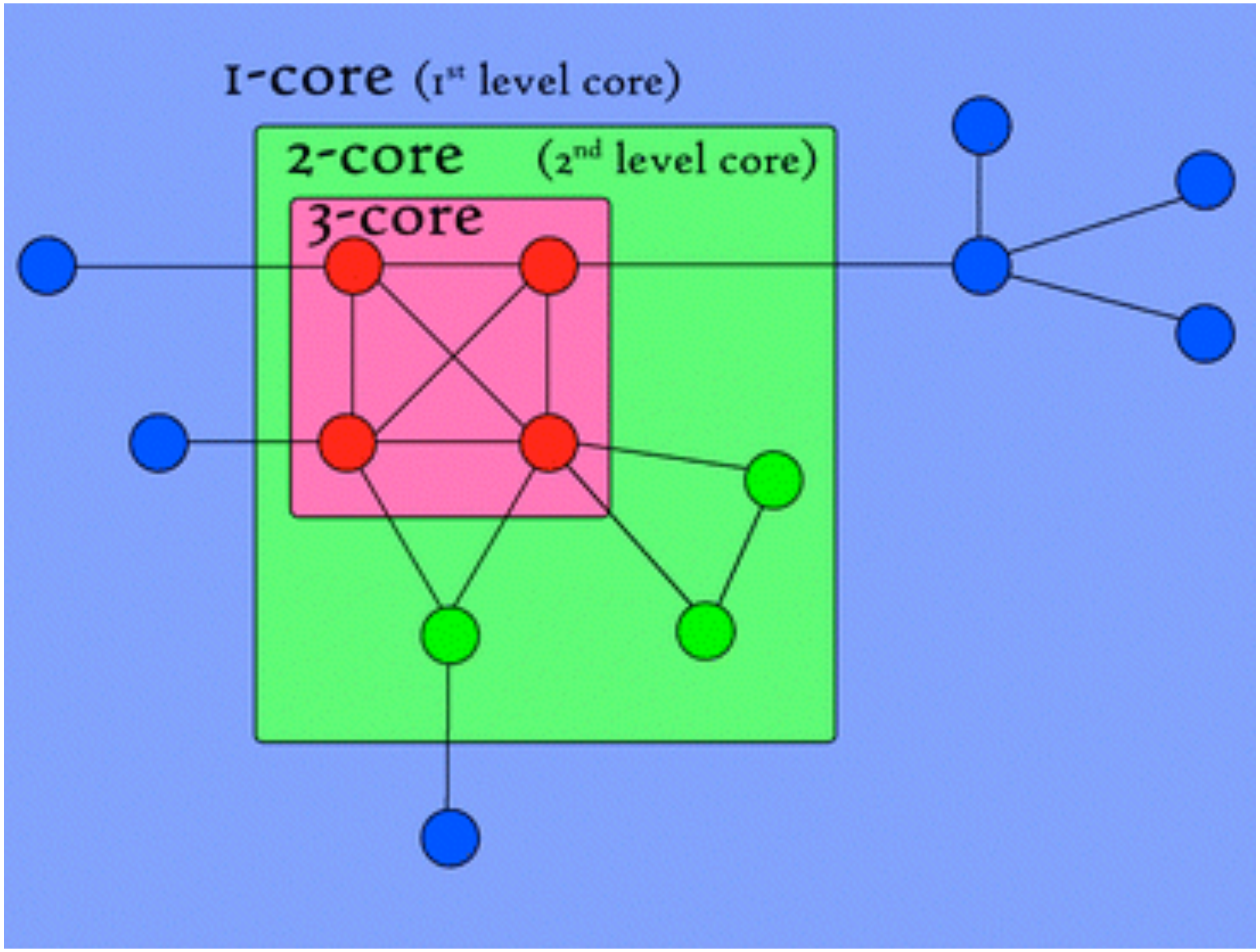


K_5

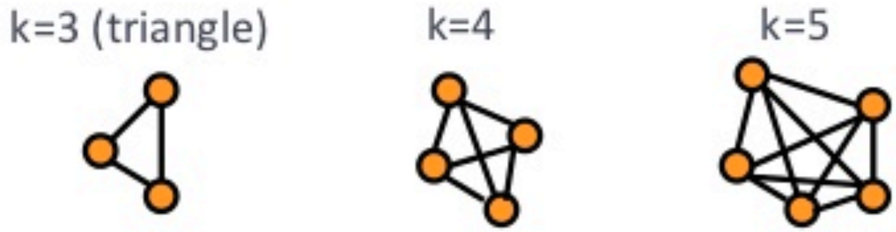


K_8

The k-core and k-shell

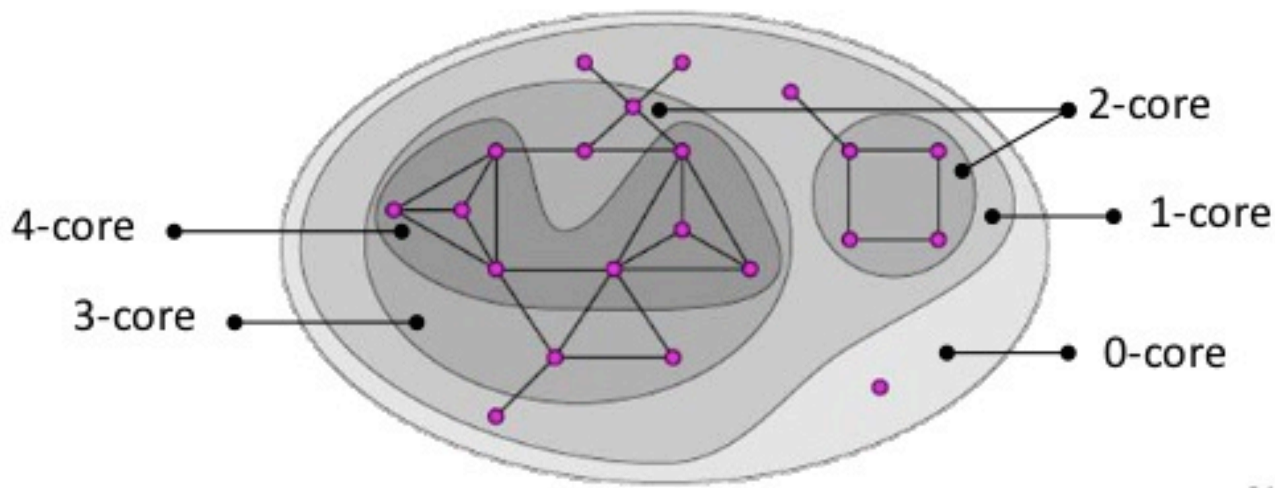


- k-clique



- N-clique

- k-core



k-core decomposition

- For visualization

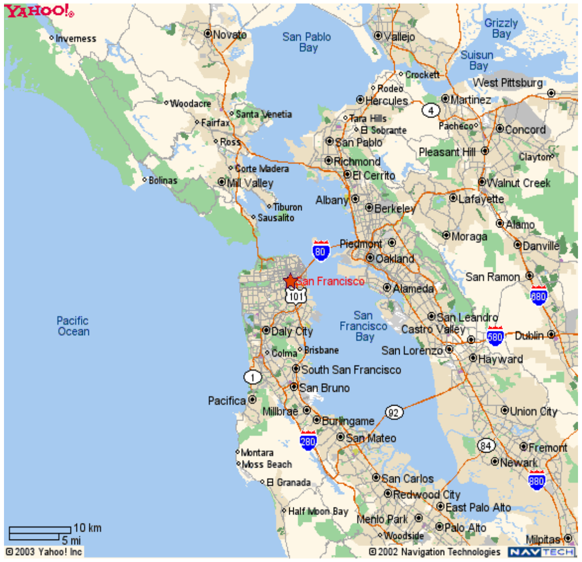
- k-core decomposition of the Internet
  - router level
  - AS level
  - e.g. Carmi et. al. PNAS 2007.

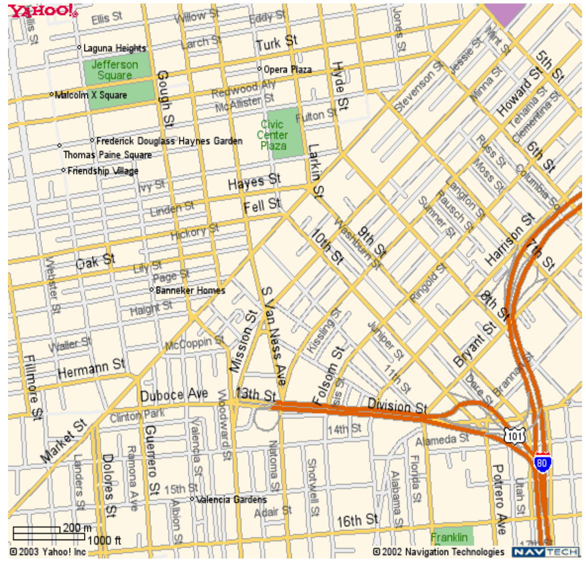
  "A model of Internet topology using k-shell decomposition"
  A nucleus, a fractal layer, and tendrils.

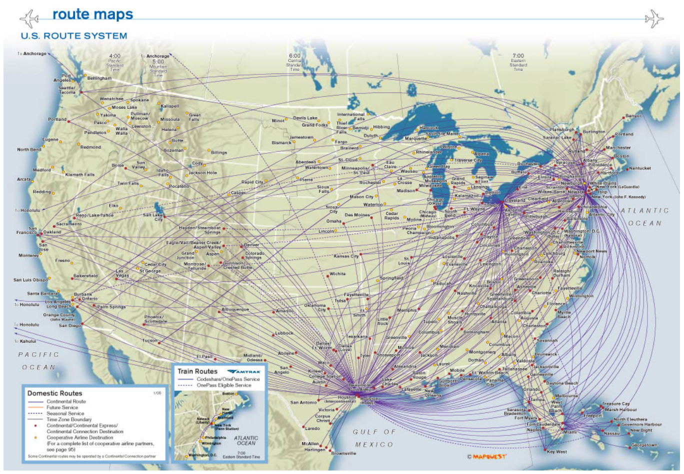
- in random graphs and statistical physics:

  "K-core organization of complex networks"
  SN Dorogovtsev, AV Goltsev, JFF Mendes
  Physical review letters, 2006.

Topic 2: Flows on spatial networks







Topics

- Optimal allocation of facilities and transport networks:
  - Michael Gastner (SFI) and Mark Newman (U Mich)

- Network flows on road networks
  - I. User vs System Optimal
  - II. Braess' Paradox
  - Michael Zhang (UC Davis)

- Layered interacting networks:
  - Kurant and Thiran, PRL 2006.
  - Buldyrev et al, Nature 2010.
  - etc.

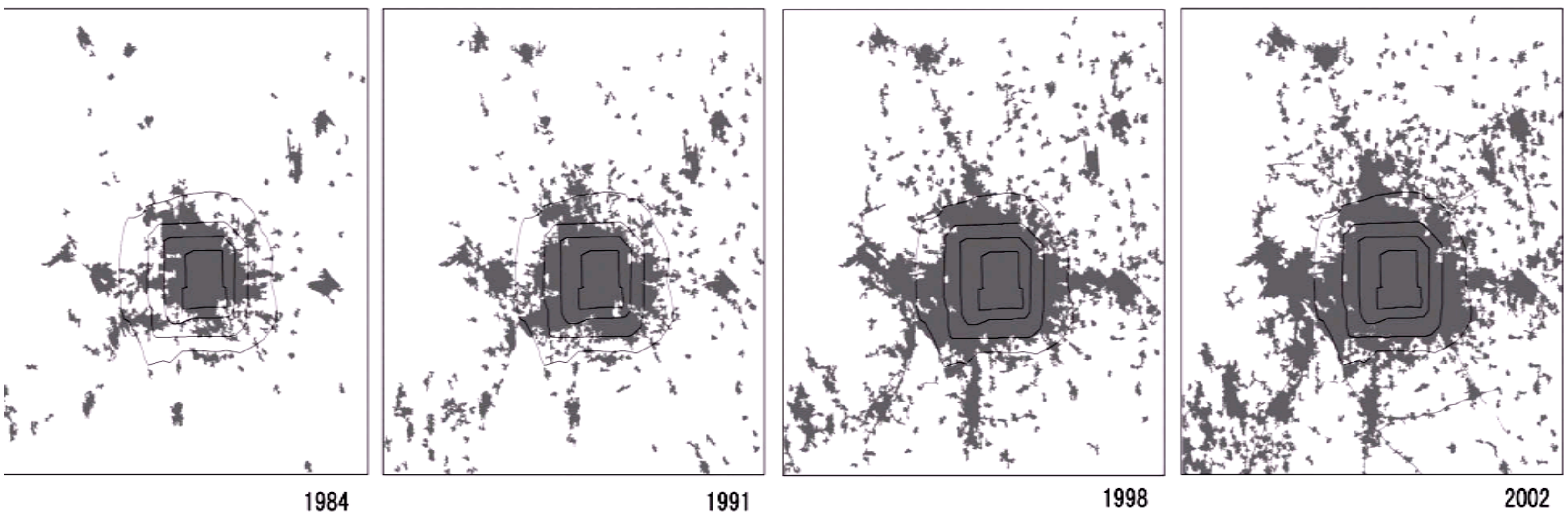
Optimal design of spatial distribution systems:

(Download: Gastner.pdf)

Flow on transportation networks:

(Download: Zhang.ppt)

Growing the highway network (Beijing)



北京城市“摊大饼”——根据 1984—2002 年卫星遥感图像制作的北京城市中心区蔓延示意图

Network flow solvers, e.g., CPLEX
(Operations research solution for optimal flow)

Must know a priori:

- All source destinations pairs
- Total demand between all pairs
- Capacity of the lines

Nash equilibrium versus System optimal

Prisoner's Dilemma

	Cooperate	Defect
Cooperate	3, 3	0, 5
Defect	5, 0	1, 1

- Blue Cooperates/Red Cooperates — Blue gets payout "3"
- Blue Cooperates/Red Defects – Blue gets "0"
- Blue Defects/Red Defects – Blue gets "1"
- Blue Defects/Red Cooperates – Blue gets "5"

– Average expected payout for defect is "3", for cooperate is "1.5". **Blue always chooses to Defect!** Likewise Red always chooses Defect.

– Both defect and get "1" (Nash), even though each would get a higher payout of "3" if they cooperated (Pareto efficient).

User optimal versus system optimal

Act on self interests (User Equilibrium):

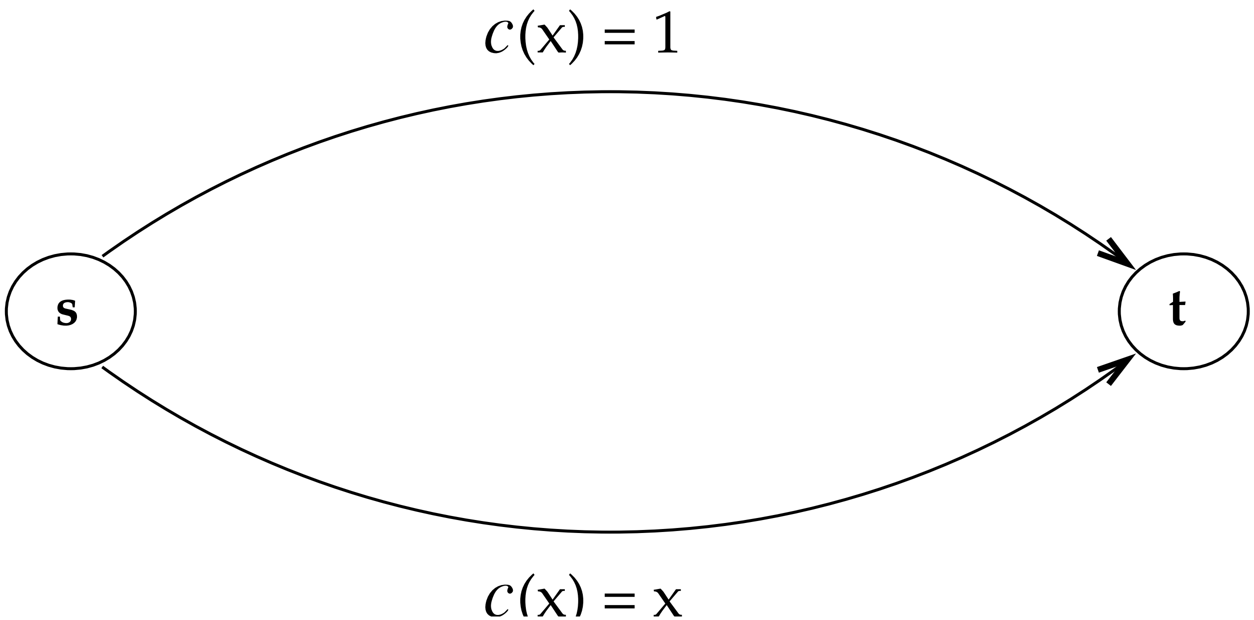
- Travelers have full knowledge of the network and its traffic conditions
- Each traveler minimizes his/her own travel cost (time)

Act on public interests (System Optimal):

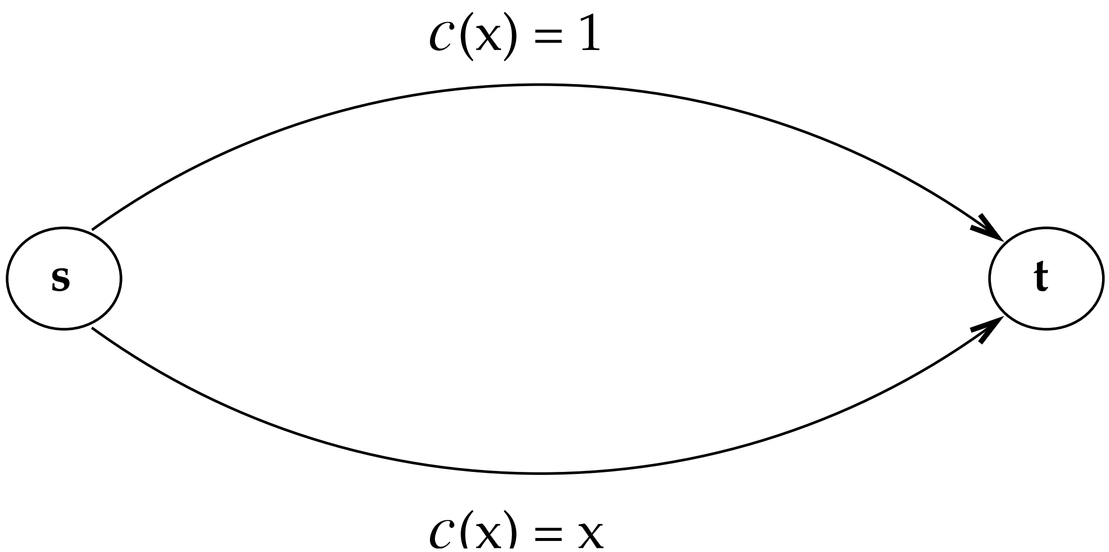
- Travelers choose routes to make the total travel time of all travelers minimal (which can be achieved through choosing the routes with minimal marginal travel cost)

$$\min \sum_{a \in A} t_a(v_a) v_a$$

Pigou's example: User versus system optimal



- Two roads connecting source, s, and destination, t
- Top road, "infinite" capacity but circuitous; 1 hour travel time
- Bottom road, direct but easily congested; travel time equal to the fraction of traffic on the route



- Everyone takes the bottom road!
  - It is never worse than the top road, and sometimes better
  - Average travel time = 1 hour = 60 mins

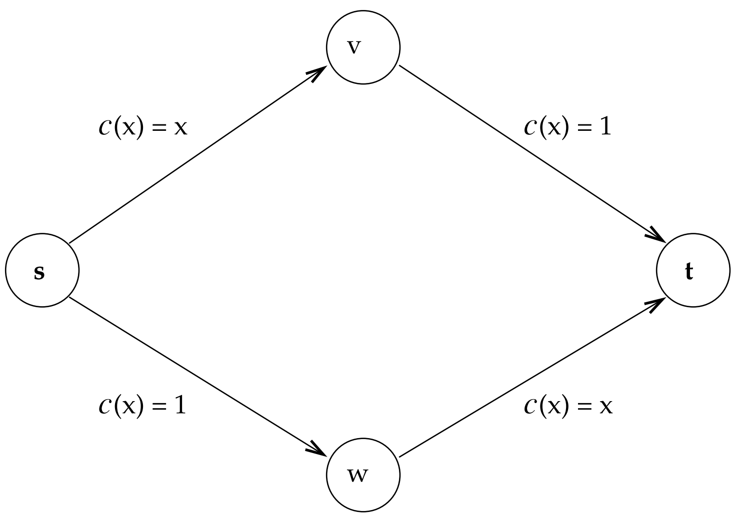
- If could incentivize half the people to take the upper road, then lower road costs 30 mins.
  - Average travel time: 0.5*60mins + 0.5*30 mins = 45 mins!

Braess Paradox

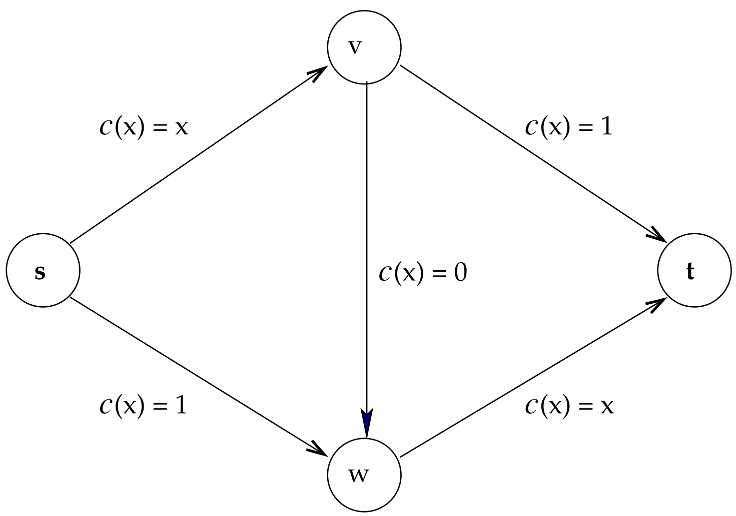
- Dietrich Braess, 1968

  (Braess currently Prof of Math at Ruhr University Bochum, Germany)

- In a user-optimized network, when a new link is added, the change in equilibrium flows might result in a higher cost, implying that users were better off without that link.



(a) Initial network



(b) Augmented network

- Recall Zhang notation
  - q_{ij} is overall traffic demand from node i to j.
  - $t_a(\nu_a)$ is travel cost along link a,
  - which is a function of total flow that link ν_a.
- Equilibrium is when the cost on all feasible paths is equal

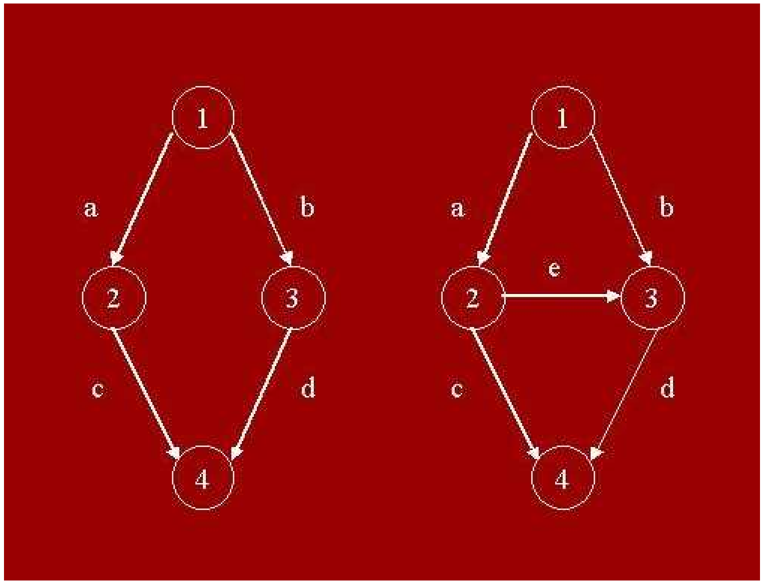
Getting from 1 to 4

Assume traffic demand $q_{14} = 6$. Originally 2 paths (a-c) and (b-d).

- $t_a(\nu_a) = 10\nu_a$
- $t_b(\nu_b) = \nu_b + 50$
- $t_c(\nu_c) = \nu_c + 50$
- $t_d(\nu_d) = 10\nu_d$

$\Longrightarrow$ Eqm: $\nu = 3$ on each link

$\mathbf{C_1 = C_2 = 83}$



Add new link with $t_e(\nu_e) = \nu_e + 10$

Now three paths:

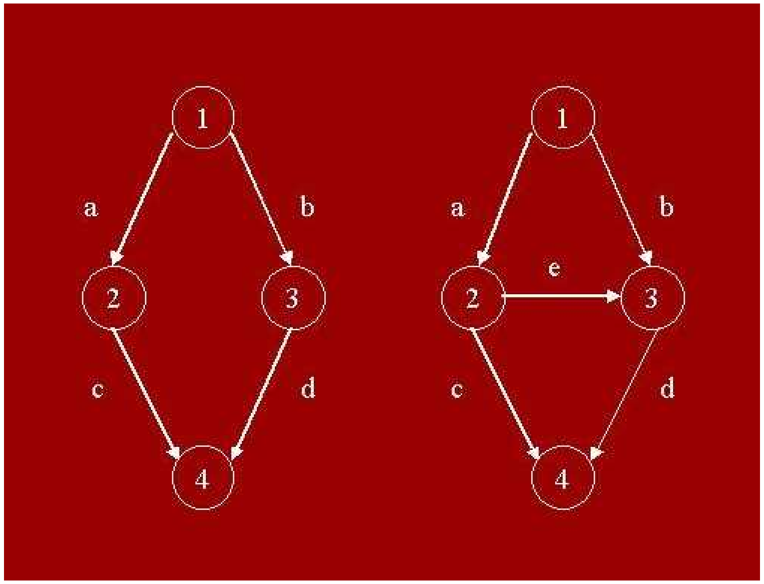
Path 3 (a - e - d), with $\nu_e = 0$ initially, so $C_3 = 0 + 10 + 0 = 10$

$C_3 < C_2$ and C_1 so new equilibrium needed.

- By inspection, shift one unit of flow form path 1 and from 2 respectively to path 3.

- Now all paths have flow $f_1 = f_2 = f_3 = 2$.

- Link flow $\nu_a = 4$, $\nu_b = 2$, $\nu_c = 2$, $\nu_d = 4$, $\nu_e = 2$.



$t_a = 40$, $t_b = 52$, $t_c = 52$, $t_d = 40$, $t_e = 12$.

$C_1 = t_a + t_c = \mathbf{92}$; $C_2 = t_b + t_d = \mathbf{92}$; $C_3 = t_a + t_e + t_d = \mathbf{92}$.

- $92 > 83$ **so just increased the travel cost!**

Braess paradox – Real-world examples

(from http://supernet.som.umass.edu/facts/braess.html)

- 42nd street closed in New York City. Instead of the predicted traffic gridlock, traffic flow actually improved.

- A new road was constructed in Stuttgart, Germany, traffic flow worsened and only improved after the road was torn up.

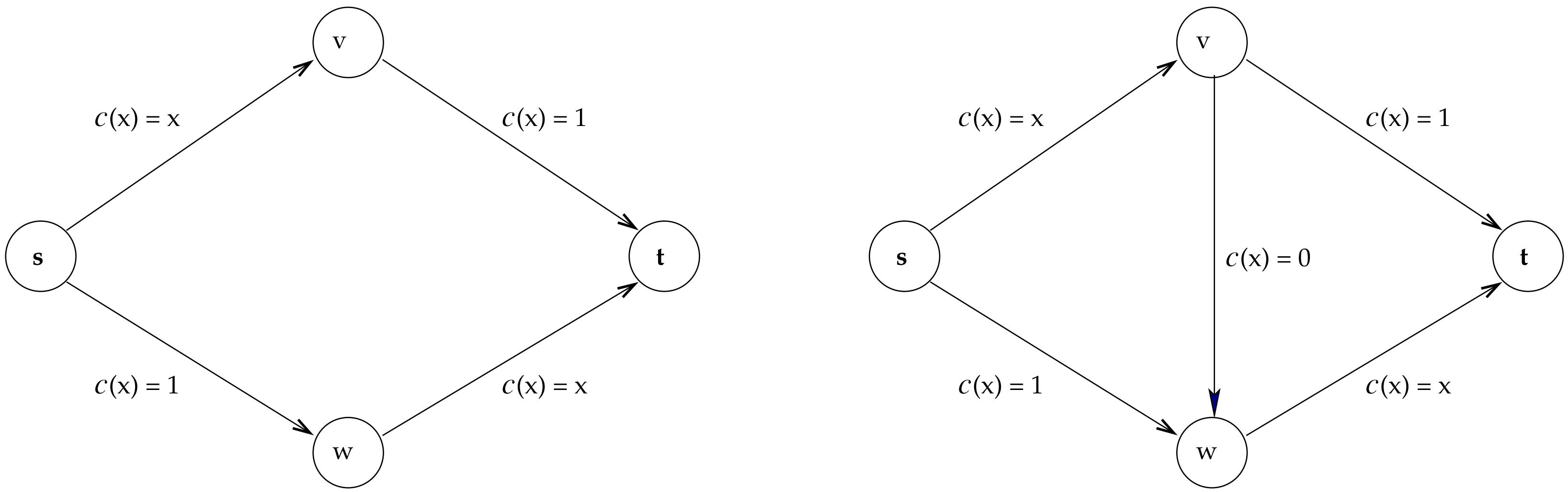
Braess paradox depends on parameter choices

- "Classic" 4-node Braess construction relies on details of q_{14} and the link travel cost functions, t_i.

- The example works because for small overall demand (q_{14}), links a and d are cheap. The new link e allows a path connecting them.

- If instead demand large, e.g. $q_{14} = 60$, now links a and d are costly! ($t_a = t_d = 600$ while $t_b = t_c = 110$). The new path a-e-d will always be more expensive so $\nu_e = 0$. No traffic will flow on that link. So Braess paradox does not arise for this choice of parameters.

Another example of Braess



(a) Initial network (b) Augmented network

How to avoid Braess?

- Back to Zhang presentation .... typically solve for optimal flows numerically using computers. Can test for a range of choices of traffic demand and link costs.

More flows and equilibirum

- David Aldous, "Spatial Transportation Networks with Transfer Costs: Asymptotic Optimality of Hub and Spoke Models"

- Marc Barthélemy, "Spatial networks" *Physics Reports* 499 (1), 2011.

- Flows of material goods, self-organization:
  Helbing et al.

- Jamming and flow (phase transitions):
  Nishinari, Liu, Chayes, Zechina.

- Algorithmic game theory: Multiplayer games for users connected in a network / interacting via a network.
  - Designing algorithms with desirable Nash equilibrium.
  - Computing equilibrium when agents connected in a network.