Glider and airplane design for students

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Abstract: Design is an open ended process driven by the mission requirements. In this paper, we account for the competition rules by including those requirements in different ways and with different levels of modelisation. The first step is achieved with a ‘cookie cutter’ approach in the rapid prototyping code. This is followed by a more accurate estimation of the take-off velocity with total airplane weight, which will be matched with the longitudinal equilibrium capabilities of the complete aircraft. In this process, the location of the centre of gravity is found. The maximum payload is also obtained. With engine-off, the glider can be trimmed for maximum distance or maximum duration. Winglet design is also discussed. Finally, the design of the classic configuration with a lifting tail at take-off is explained, which, with the double-element airfoil, have been features of the UC Davis entries for the last few years.

Keywords: glider design; airplane design; student design competition; aerodynamic configuration; double-element airfoil; winglet; longitudinal stability; lifting tail.

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Biographical notes: Jean-Jacques Chattot received his PhD in Aeronautical Sciences from the University of California Berkeley in 1971. He worked in the aerospace industry in France before joining, in 1989, the Department of Mechanical and Aerospace Engineering at UC Davis. Trained in computational fluid dynamics (CFD), he has been conducting research in transonic flow, applied aircraft aerodynamics and wind turbine aeroelastic simulation.

1 Introduction

This paper deals with simple aspects of airplane design and stability analysis which have been motivated by the author’s participation over the last 15 years, as faculty advisor of a team of undergraduate students, in the Society of Automotive Engineering (SAE) Aero-Design West competition. This is a heavy lifter competition. Each year SAE publishes new rules for the competition, enticing students to design a different airplane. In the ‘regular class’, the powerplant and engine fuel are imposed and the airplane must take-off within a 200 ft runway. Some other constraints change from year to year, such as maximum wing span, or maximum lifting surfaces area, etc. In the ‘open class’, the engine displacement is specified, but the engine make and fuel
are the team’s choice. A take-off distance is either imposed or targeted with a flight raw score depending heavily on how close to 200 ft take-off occurs. A round counts when the airplane lands within a 400 ft marked area of the runway without having lost any part. Then, the payload is removed from the payload bay and is weighted to attribute points to the flight. During these years, basic prediction tools were developed to help students design and predict their airplane performance. They consist of three simple computer models, a rapid prototyping code, a ground acceleration code and a flight equilibrium code. Furthermore, each year, the SAE competition has served as an ongoing project in the applied aerodynamics class, culminating in a design project, as last assignment for the quarter, that adheres to the SAE rules of that year. This was a source of motivation for the students as many of the team members of the Aggie Micro Aeronautics Team (AMAT) are taking the class. Since 2006, a double element wing was used, which attracted a lot of interest and proved to produce a very high lift coefficient.

This paper describes the various models implemented, with a view to helping students with design and estimation of airplane performance. First, the aerodynamic model of a classical configuration is developed, with the main wing in the front and the tail behind it. This is an easier model to study than the ‘canard’ configuration in which small lifting surfaces are placed near the front of the fuselage and the main wing is moved further back, due to the more complex interaction of the canard on the main wing.

2 Model of a classical wing/tail configuration

As starting point, one shall assume incompressible, steady, attached flow on the fuselage and the tail. The main wing, of medium to large aspect ratio, either uses the simple linear model presented below, or a more accurate model based on Prandtl lifting line theory. When the latter model is not available, the linear model for the wing is used.

2.1 Linear model for the main wing

Consider a rectangular, untwisted wing of span \( b_m \) and chord \( c_m \), equipped with an airfoil of constant relative camber, \( d_m/c_m \). The wing aspect ratio is \( AR_m = b_m/c_m \). Let \( \alpha \) be the geometric incidence, i.e., the angle between the incoming flow vector \( \vec{U} \) and the fuselage axis \( \vec{Ox}_1 \) which serves as reference for the angles. The coordinate system attached to the airplane is composed of the \( (\vec{Ox}_1, \vec{Oy}_1, \vec{Oz}_1) \) axes, with \( \vec{Ox}_1 \) oriented downstream, \( \vec{Oy}_1 \) aligned with the right wing and \( \vec{Oz}_1 \) completing the direct coordinate frame. Accounting for the induced incidence, assumed constant along the span (elliptic loading), and building up on the results from thin airfoil theory, the lift coefficient is given by:

\[
C_{Lm} = \frac{2\pi}{1 + \frac{2}{AR_m}} \left( \alpha + t_m + 2 \frac{d_m}{c_m} \right)
\]

(1)

\( t_m \) is the setting angle of the wing on the fuselage. The linear model comprises the lift slope and the \( \alpha = 0 \) lift coefficient as:

\[
\frac{dC_{Lm}}{d\alpha} = \frac{2\pi}{1 + \frac{2}{AR_m}}, \quad C_{Lm0} = \frac{2\pi}{1 + \frac{2}{AR_m}} \left( t_m + 2 \frac{d_m}{c_m} \right)
\]

(2)
The reference area for the wing lift is the wing area $A_m = b_m c_m$. At small incidences, the friction drag is estimated with a flat plate formula:

$$C_{Dm0} = \begin{cases} \frac{2}{\sqrt{\text{Re}_{cm}}} \frac{1.328}{2}, & \text{Re}_{cm} < 5 \times 10^5 \\ \frac{2}{\sqrt{\text{Re}_{c_m}}}, & \text{Re}_{cm} > 5 \times 10^5 \end{cases}$$

where $\text{Re}_{cm}$ is the Reynolds number based on the wing chord. The factor 2 accounts for the two sides of the wing. The reference area for the wing friction drag is $A_m$.

The induced drag is given by the classical formula:

$$C_{Dim} = \frac{C_{Lm}^2}{\pi e_m A R_{m}}$$

$e_m$ is the Oswald efficiency factor. For a medium aspect ratio wing ($AR \simeq 10$) a value $e_m = 0.9$ is acceptable, but values that are higher or even larger than one are possible if winglets are added. The total drag of the main wing is thus $C_{Dm} = C_{Dm0} + C_{Dim}$.

The moment coefficient of the main wing about the origin of the coordinate system, located at the nose of the airplane, results also from thin airfoil theory as:

$$C_{M,om} = C_{M,acm} - \frac{x_{acm}}{c_{am}} C_{Lm} = -\frac{d_m}{e_m} \frac{x_{acm}}{c_{am}} \left( \frac{dC_{Lm}}{d\alpha} \alpha + C_{Lm0} \right)$$

where $C_{M,acm} = -\pi \frac{d_m}{c_m}$ is the moment about the aerodynamic center of the wing (function of the mean camber), $x_{acm}$ is the location of the quarter-chord of the wing and $c_{am}$ is the mean aerodynamic chord (here $c_{am} = c_m$). The linear decomposition of the moment reads:

$$\frac{dC_{M,om}}{d\alpha} = -\frac{x_{acm}}{c_{am}} \frac{dC_{Lm}}{d\alpha}, \quad C_{M,om0} = -\frac{d_m}{e_m} \frac{x_{acm}}{c_{am}} C_{Lm0}$$

The reference ‘volume’ for the wing moment is $A_m c_{am}$.

### 2.2 Linear model for the tail

With the classical configuration, the tail is influenced by the downwash of the main wing. The downwash varies from $w_w$ at the wing ($x = 0$) to $w_T = 2w_w$ in the Trefftz plane ($x = +\infty$). The coefficient of downwash, $k_i = w_i / |w_w|$ at the tail location is between –1 and –2. A study of the downwash induced by the vortex sheet of an elliptically loaded wing, provides a quantification of this effect, as shown in Figure 1.
One also assumes a rectangular tail with an elliptic loading. This is a valid approximation since the tail has a low aspect ratio $AR_t = b_t/c_t$ where $b_t$ and $c_t$ are the tail span and chord, respectively. The tail mean camber is $d_t$. The tail lift coefficient is given by:

$$C_{Lt} = 2\pi \left( \alpha + \alpha_{im} + \alpha_{it} + t_t + 2 \frac{d_t}{c_t} \right)$$  \hspace{1cm} (7)$$

where $\alpha_{im}$ represents the induced incidence due to the wing downwash at the tail lifting line and $\alpha_{it}$ that due to the tail downwash on itself. Using the results of Prandtl lifting line:

$$\alpha_{im} = k_i \frac{|w_w|}{U} = k_i \frac{C_{Lm}}{\pi AR_m}, \quad \alpha_{it} = \frac{w_t}{U} = -\frac{C_{Lt}}{\pi AR_t}$$  \hspace{1cm} (8)$$

Hence, solving for $C_{Lt}$ gives:

$$C_{Lt} = \frac{2\pi}{1 + \frac{1}{\pi AR_t}} \left( \alpha + \frac{k_i}{\pi AR_m} \left( \frac{dC_{Lm}}{d\alpha} \alpha + C_{Lm0} \right) + t_t + 2 \frac{d_t}{c_t} \right)$$  \hspace{1cm} (9)$$

The linear decomposition for the tail lift consists of:

$$\frac{dC_{Lt}}{d\alpha} = \frac{2\pi}{1 + \frac{1}{\pi AR_t}} \left( 1 + \frac{k_i}{\pi AR_m} \frac{dC_{Lm}}{d\alpha} \right),$$  \hspace{1cm} (10)$$

$$C_{Lt0}(t_t) = \frac{2\pi}{1 + \frac{1}{\pi AR_t}} \left( t_t + 2 \frac{d_t}{c_t} + \frac{k_i}{\pi AR_m} C_{Lm0} \right)$$

Note that the tail setting angle $t_t$, which controls the airplane flight operation, does not appear in the tail lift slope. The reference area for the tail lift is the tail area $A_t$.

The tail drag is the sum of the friction drag $C_{Df0}$ and the induced drag $C_{Di0}$ calculated with formula similar to those used for the wing. The area of reference for the tail drag is $A_t$. 

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**Figure 1** Coefficient of downwash as a function of reduced distance from the main wing
The moment coefficient for the tail can be decomposed as:

\[
\frac{dC_{M,t}}{d\alpha} = -\frac{x_{act}}{c_t} \frac{dC_L}{d\alpha}, \quad C_{M,t0}(t_t) = -\pi \frac{d_t}{c_t} - \frac{x_{act}}{c_t} C_{L,t0}(t_t)
\]  

Again, we note that the moment slope for the tail is independent of the tail setting angle.

### 2.3 Aerodynamic coefficients for the fuselage

According to slender body theory, the fuselage does not contribute to lift.

The drag coefficient, \(C_{D,f0}\), is calculated with the flat plate formula at Reynolds number \(Re_f = \rho U l_{ref}/\mu\), where \(l_{ref}\) is the fuselage length. The plate width is estimated to be \(h_f\) such that the fuselage wetted area is approximately \(h_f l_{ref}\).

**Figure 2** Moment of a body of revolution at incidence

The fuselage moment coefficient is now obtained. In slender body theory, the lift of a truncated body of revolution, with base area \(A = \pi R^2\) as reference area, is given by:

\[
C_L = 2\alpha
\]  

A truncated fuselage that extends from the nose to \(x\) will have a lift force given by:

\[
L = \frac{1}{2} \rho U^2 A(x) 2\alpha
\]  

A small section of the fuselage from \(x\) to \(x + dx\) will have a lift force \(dL\) (see Figure 2):

\[
dL = \rho U^2 \alpha A'(x) dx
\]  

that will contribute to the pitching moment as:

\[
dM_{of} = -x dL = -\rho U^2 \alpha A'(x) dx
\]

The minus sign is due to the convention of a positive nose up moment. The integration yields:

\[
M_{of} = -\rho U^2 \alpha \int_0^{l_{ref}} A'(x) dx
\]

\[
= -\rho U^2 \alpha \left\{ [A(x)x]_{0}^{l_{ref}} - \int_0^{l_{ref}} A(x) dx \right\}
\]

In the integration by parts, the first term cancels out since the fuselage has zero area at its extremities, and the integral term represents the fuselage volume $\vartheta_f$. Hence the fuselage moment reduces to:

$$M_{o,f} = \rho U^2 \vartheta_f \alpha$$

In dimensionless form, using the maximum cross section area $A_f$ as reference area, one gets:

$$C_{M,o,f} = \frac{2 \vartheta_f}{A_f l_{ref}} \alpha$$

Note that the fuselage moment slope $dC_{M,o,f}/d\alpha$ is positive, which is a distabilising moment. In general, the fuselage contribution to the moment is small.

### 2.4 Global aerodynamic coefficients for the glider

The aerodynamic coefficients for the complete configuration cannot be obtained by summing the individual coefficients of each component of the glider, since the reference areas and reference lengths are different. But forces and moments can be summed, and not accounting for the dynamic pressure, one can write:

$$A_{ref} C_L = A_m C_{Lm} + A_t C_{Lt}$$

$$A_{ref} C_D = A_m C_{Dm} + A_t C_{Dt} + A_f C_{Df}$$

$$A_{ref} l_{ref} C_{M,o} = A_m c_{am} C_{M,om} + A_t c_{at} C_{M,ot} + A_f l_{ref} C_{M,o,f}$$

The global aerodynamic model for the lift and moment coefficients, is linear. If one chooses as area of reference the sum of the wing and tail areas, the global lift coefficient becomes:

$$C_L = \frac{A_m C_{Lm} + A_t C_{Lt}}{A_m + A_t}$$

and the global moment coefficient:

$$C_{M,o} = \frac{A_m c_{am} C_{M,om} + A_t c_{at} C_{M,ot} + A_f l_{ref} C_{M,o,f}}{(A_m + A_t)l_{ref}}$$

It is easy to see that the lift and moment slopes are thus combinations of the wing, tail and fuselage slopes and do not depend on the tail setting angle $t_t$. But the $\alpha = 0$ coefficients of lift and moment depend on $t_t$. 
3 Rapid prototyping

Rapid prototyping is a process by which the airplane main lifting component, the wing, can be sized and its weight estimated. For a composite wing, given the span and the chord, the weight depends on its volume, and the density of the foam used to build the wing core, and its surface area, and the density of the carbon fiber/glass fiber materials to make the wing skin. The weight of the other components, fuselage, tail and landing gear are also estimated, and by subtraction from the gross weight at take-off, the payload prediction is obtained and a flight raw score calculated. This code is a ‘cookie-cutter’ for the wing as it estimates the best chord size for a given span and wing profile lift characteristics. The following assumptions are made to simplify as much as possible the analysis:

1. rectangular wing with span $b$ and chord $c$
2. elliptic circulation (the lower the aspect ratio $AR$, the more appropriate it is)
3. drag coefficient defined by

$$C_D = C_{D0} + \frac{C_L^2}{\pi e AR} \quad (24)$$

4. engine thrust $T(V) = T_0 - KV$
5. rolling friction is neglected (although it had been implemented when taking-off from grass field),

The aspect ratio is defined as $AR = b^2/S$ where $S$ is the wing area and reduces to $AR = b/c$ for a rectangular wing. The zero-lift drag coefficient $C_{D0}$ is estimated from the airplane wetted area and parasitic drags (landing gear primarily). The Oswald efficiency factor, $e$, is close to unity ($e \approx 0.9$) for a wing of medium aspect ratio, $AR = 10$ or so, but could be closer or even larger than one with winglets. The lift coefficient $C_L$ depends on the setting angle of the wing on the fuselage which determines the wing incidence during the rolling phase. The propeller is selected to match the engine and to produce a high static thrust $T_0$ in $(N)$ at high nominal rpm, measured with a fish scale. As the airplane accelerates, the thrust decreases. For simplicity, we use the actuator disk theory to estimate $T(V)$ at low speeds. A more accurate approach would be to measure the thrust of the propeller placed in a wind tunnel or to use a propeller code (if the engine power were known as a function of rpm). Let $U_b$ be the average axial induced velocity at the rotor, then, according to the actuator disk theory, power and thrust are given by:

$$P = 2\pi \rho R^2 (V + U_b)^2 U_b \quad (25)$$

$$T = 2\pi \rho R^2 (V + U_b) U_b, \quad \Rightarrow T_0 = 2\pi \rho R^2 U_b^2 \quad (26)$$

Assuming that the power remains constant at low speeds, the condition $dP = 0$ reduces to:

$$\frac{dU_b}{dV} = -\frac{2U_b}{V + 3U_b} \quad (27)$$
On the other hand, the change in thrust at low speeds reads:

\[
\frac{dT}{dV} = 2\pi \rho R^2 \left( U_b + (V + 2U_b) \frac{dU_b}{dV} \right)
\]

\[
= -2\pi \rho R^2 \frac{V + U_b}{V + 3U_b} U_b \simeq -\frac{2\pi R^2}{3} U_b
\]

(28)

Upon elimination of \(U_b\) one finds:

\[
K = \frac{dT}{dV} = \frac{\sqrt{2\pi}}{3} R \sqrt{\rho T_0}
\]

(29)

With \(T_0\) and \(K\), the model can be further developed. Newton’s second law reads:

\[
M \frac{d^2x}{dt^2} = M \frac{dV}{dt} = MV \frac{dV}{dx} = T_0 - KV - \frac{1}{2} \rho V^2 A_{ref} C_D
\]

(30)

The last term corresponds to the total aerodynamic drag, i.e., viscous drag and induced drag, \(D = \frac{1}{2} \rho V^2 A_{ref} C_D\). One can separate the variables and write:

\[
\frac{MV dV}{T_0 - KV - \frac{1}{2} \rho A_{ref} C_D V^2} = dx
\]

(31)

Let \(V_1\) and \(V_2\) be the roots of the quadratic equation for \(V\):

\[
V_{1,2} = -K \pm \sqrt{K^2 + 2\rho A_{ref} C_D T_0 \rho A_{ref} C_D}
\]

(32)

The positive root, \(V_2\), corresponds to the maximum velocity obtained as asymptotic limit over an infinite rolling distance (\(V_1\) has no physical significance). The equation can now be written as:

\[
V dV = \frac{1}{V_1 - V_2} \left( \frac{V_1}{V - V_1} - \frac{V_2}{V - V_2} \right) dV
\]

\[
= -\frac{1}{2} \rho A_{ref} C_D dx
\]

(33)

Upon integration from \(V = 0\) for a finite rolling distance (say, \(L = 55 m = 180 ft\) to allow for rotation before take-off) one finds:

\[
\frac{1}{V_1 - V_2} \left( V_2 \ln \left( \frac{V_2 - V}{V_2} \right) - V_1 \ln \left( \frac{V - V_1}{-V_1} \right) \right) = \frac{1}{2} \rho V^2 A_{ref} C_D \frac{L}{M}
\]

(34)

This is an implicit equation for the take-off velocity \(V\). It can be solved iteratively as:

\[
V^{(n+1)} = V_2 \left[ 1 - \left( \frac{-V_1}{V^{(n)} - V_1} \right)^\frac{L}{MV_2} \right] \exp \left\{ - \frac{L}{MV_2} \sqrt{K^2 + 2\rho A_{ref} C_D T_0} \right\}
\]

(35)
where $n = 0, 1, \ldots$ is the iteration index. An initial guess is $V^{(0)} = V_2$.

Given a maximum lift coefficient $C_{L_{\text{max}}}$ for the wing at take-off, the drag $D$ at take-off is known and the climb angle $\beta$ can be calculated as:

$$\beta = \frac{T(V) - D}{Mg}$$  \hspace{1cm} (36)

The wing span being given (and changed if the rules allow), the code performs a loop on the wing chord by steps of 1 cm. For each chord size, the wing weight is calculated and the gross weight of the airplane estimated. The take-off velocity is then calculated, using the above iterative formula. Finally, the climb angle is obtained and the loop on the chord is terminated when the climb angle is $\beta \geq 3\,\text{deg}$. This value is chosen to allow the airplane to clear the runway, but also to account for uncertainties. The flight score is predicted from the payload.

Fine tuning of the configuration is carried out with the next level of modelisation.

4 Acceleration phase

The acceleration phase is very critical as it determines the take-off velocity $V_{t,o}$ at the point where the pilot rotates the airplane to give it the incidence needed to begin climbing. The velocity $V$ depends primarily on the powerplant and the mass of the airplane. To a lesser extent it depends on the parasitic drags, aerodynamic and rolling friction drags. This is the reason why a high wing lift coefficient $C_{L_{\text{max}}}$ is needed in order to lift the highest possible weight. These can be modelled in more details with the acceleration code as:

$$M \frac{d^2x}{dt^2} = T \left[ \frac{dx}{dt} \right] - \frac{1}{2} \rho \left( \frac{dx}{dt} \right)^2 A_{\text{ref}}(C_{D0} + C_{Dl}) - a_k W_{\text{app}}$$  \hspace{1cm} (37)

In this formulation, the thrust $T[V]$ can be a more elaborate function of the velocity, say from wind tunnel measurements, although the previous linear model can be used when more detailed data is not available. The zero-lift drag $C_{D0}$ will vary with velocity via the Reynolds number $R_e = \rho V l_{\text{ref}} / \mu$ based on the air density and dynamic viscosity, the velocity and the fuselage length $l_{\text{ref}}$. In general, we will assume turbulent flow on most of the wetted area of the airplane. Therefore the viscous drag will be estimated as

$$C_{D0}(R_e) = C_{D_{\text{ref}}} \left( \frac{V_{\text{ref}}}{V} \right)^{\frac{1}{2}}$$  \hspace{1cm} (38)

The reference drag coefficient, $C_{D_{\text{ref}}}$ is estimated using the friction drag of each streamlined component and tables for blunt elements such as the landing gear of the airplane. The reference velocity is chosen typically as $V_{\text{ref}} = 20\,\text{m/s}$. Accounting for the main wing, the tail, the fuselage and the landing gear, adding the different contributions with their corresponding reference areas, gives:

$$A_{\text{ref}} C_{D_{\text{ref}}} = A_m C_{Dm} + A_t C_{Dt} + A_f C_{Df} + A_g C_{Dg}$$  \hspace{1cm} (39)
The induced drag is primarily due to the main wing because of the high lift coefficient, even during the rolling phase. The induced drag of the tail is neglected. The induced drag reads as previously:

$$C_{Di} = \frac{C_L^2}{\pi e AR}$$  \hspace{1cm} (40)

The rolling friction is modelled with a coefficient of friction \(a_k\) as:

$$F_r = a_k W_{app}$$  \hspace{1cm} (41)

\(a_k\) depends on the materials in contact and for rubber on asphalt a value \(a_k = 0.03\) is used. The normal force acting on the tires is the apparent weight of the airplane, where:

$$W_{app} = Mg - \frac{1}{2} \rho V^2 A_{ref} C_L$$  \hspace{1cm} (42)

This model is transformed into a system of two first-order ordinary differential equations in two unknowns \(x\) and \(V\) as:

$$\begin{cases}
\frac{dx}{dt} = V \\
\frac{dV}{dt} = \left(T[V] - \frac{1}{2} \rho V^2 A_{ref} (C_{D0}[V] + C_{Di}) - a_k W_{app}[V]\right) / M
\end{cases}$$  \hspace{1cm} (43)

Integration is carried out with a fourth-order Runge-Kutta scheme and a time step \(\Delta t = 0.01\) s. The total mass \(M\) is varied and a relation \(V_{t.o.}(M)\) is obtained for the take-off velocity achievable for different airplane masses. This result is important as it will determine the feasibility of taking-off and flying a round when running the equilibrium model. For an open class entry, the results are presented in Figure 3.

**Figure 3** Take-off velocity vs. total airplane mass
5 Longitudinal equilibrium

In this model, we consider the airplane flying at uniform velocity along a straight trajectory contained in a vertical plane. This corresponds to a steady situation, with no acceleration. The roll and yaw are both zero. Two coordinate systems are introduced: the aerodynamic coordinate system in which the \( x \)-axis is aligned with the airplane velocity vector but oriented in the opposite direction and the \( z \)-axis upward; the other coordinate system is attached to the airplane, with the \( x_1 \)-axis along the fuselage axis and the \( z_1 \)-axis upward. The origin of the coordinate systems is placed at the nose of the airplane, see Figure 4.

Figure 4 Coordinate systems for the study of longitudinal equilibrium (see online version for colours)

![Coordinate systems](image)

The incidence angle is the angle \( \alpha = (Ox, Ox_1) \), the slope angle is \( \beta = (H, Ox) \), both positive in the figure. The dimensionless coefficients, thrust, weight, lift, drag and moment coefficients are defined as:

\[
C_T = \frac{T(V)}{\frac{1}{2} \rho V^2 A_{ref}}, \quad C_W = \frac{W}{\frac{1}{2} \rho V^2 A_{ref}}, \quad C_L = \frac{L}{\frac{1}{2} \rho V^2 A_{ref}},
\]

\[
C_D = \frac{D}{\frac{1}{2} \rho V^2 A_{ref}}, \quad C_{M,o} = \frac{M_{o}}{\frac{1}{2} \rho V^2 A_{ref} l_{ref}}.
\]

The lift force includes contributions from the main wing and the tail. The latter could be positive or negative, depending on the flight conditions. Note that the fuselage does not contribute to lift, according to slender body theory. The drag is evaluated with the best estimations of zero-lift drag, using flat plate formula for the wetted areas of streamlines elements, except for the main wing, and drag tables for the landing gear. The wing viscous drag is included in the wing viscous polar: the 2D viscous polar has been obtained for a range of incidences corresponding to attached and separated flows with the well-known program XFOIL (or MSES for multi-element airfoils) of Dr. Mark Drela; the 3D polar is derived from it, using Prandtl lifting line theory and the 2D
viscous polar at each span station. The 3D polar is extended beyond stall. The induced
drags of the main wing and the tail are included as well. The moment coefficient refers
to the nose of the airplane. The geometry of the main wing double element profile is
shown in Figure 5, and the corresponding main wing 2D and 3D polars for the double
element airfoil and wing (AMAT, 2006) are shown in Figure 6 and compared with
the 2D polar of the Selig1223 airfoil at Reynolds number 200,000. It is interesting to
note the high maximum lift coefficient of the Selig 1223 with $C_{l,\text{max}} = 2.1$ at this low
Reynolds number and the even larger value achieved by the double element airfoil with
$C_{l,\text{max}} = 3.1$. Note that the 2D viscous drag of the double element is comparable to
that of the S1223 drag near maximum lift up to $C_l = 2.8$. Another noteworthy point
to make is the large induced drag of the AMAT wing which is a direct result of the
medium wing aspect ratio $AR = 8.75$, indicating that at $C_l = 2.8$, 88% of the wing
drag is induced drag.

**Figure 5** Double element geometry

![Double element geometry diagram](image)

**Figure 6** Viscous polars for the Selig 1223 airfoil and the double element airfoil and wing

![Viscous polars diagram](image)
The tail setting angle \( t_t \) is the parameter that controls the equilibrium solution (trimmed equilibrium).

The longitudinal equilibrium equations consist of two equations for the forces and one for the moment. They read:

\[
\begin{align*}
C_L + C_T \sin(\alpha + \tau) - C_W \cos \beta &= 0 \\
C_D - C_T \cos(\alpha + \tau) + C_W \sin \beta &= 0 \\
C_{M,o} + \frac{x_{eq}}{l_{ref}} C_W \cos(\alpha + \beta) &= 0
\end{align*}
\] (45)

Here, the propeller thrust makes an angle \( \tau \) with the airplane axis. There are three unknowns, the equilibrium incidence, \( \alpha_{eq} \), velocity, \( V_{eq} \) and slope, \( \beta_{eq} \). This is a highly non-linear system, especially near the stall angle (maximum lift) since multiple values of \( \alpha \) exist for a given \( C_L \). A combination of fixed-point iterations until close enough to the solution, followed by Newton’s iterations are needed to converge, when the solution exists, depending on the trim angle \( t_t \).

The equilibrium code is used concurrently with the acceleration result \( V_{t,o}(M) \) to verify the feasibility of taking-off with a given total mass \( M \). By ‘pulling on the stick’, or equivalently, by rotating the tail flap in the upward direction, the equilibrium incidence increases and the equilibrium velocity decreases. If it is possible to find a tail setting angle such that \( V_{eq} \leq V_{t,o} \), take-off can be achieved provided \( \beta \geq 3 \text{ deg} \) and the incidence is not too close to the incidence of maximum lift. If all these conditions are fulfilled, it is possible to increase the mass of the airplane by adding payload weight thus increasing the possible score.

6 Static stability

One of the most common design mistakes made by student teams is a misplacement of the center of gravity. This results in the pilot having to constantly act on the tail deflection to maintain the airplane aloft or, in the worst case, ground rolling is immediately followed by a nose up and stall at take-off with likely loss of the airplane. The study of static stability clearly demonstrates a sufficient requirement for stability. For simplicity, we use a linear model obtained from the above model by assuming the following linear behaviour of the coefficients

\[
C_L(\alpha) = \frac{dC_L}{d\alpha} \alpha + C_{L,0}(t_t), \quad C_{M,o}(\alpha) = \frac{dC_{M,o}}{d\alpha} \alpha + C_{M,o,0}(t_t)
\] (46)

Here the lift and moment slopes are assumed constant as well as the \( \alpha = 0 \) lift and moment coefficients which depend on the tail flap setting angle \( t_t \). We further assume small angles \( \alpha \) and \( \beta \) and neglect the thrust in the first equation. The system reduces to:

\[
\begin{align*}
C_L - C_W &= 0 \\
C_D - C_T + \beta C_W &= 0 \\
C_{M,o} + \frac{x_{eq}}{l_{ref}} C_W &= 0
\end{align*}
\] (47)

Substitution of \( C_W \) from the first equation into the third one yields an equation for \( \alpha_{eq} \):

\[
C_{M,o}(\alpha_{eq}) + \frac{x_{eq}}{l_{ref}} C_L(\alpha_{eq}) = C_{M,eq}(\alpha_{eq}) = 0
\] (48)
representing the moment about the center of gravity. This is easily solved with the linear model as:

\[
\alpha_{eq}(t_t) = -\frac{C_{M,ab}(t_t) + \frac{x_{cg}}{l_{ref}} C_{L0}(t_t)}{\frac{dc_{M,ab}}{dx} + \frac{x_{cg}}{l_{ref}} \frac{dc_{L}}{dx}}
\]

(49)

The equilibrium incidence is controlled by the tail setting angle.

The equilibrium velocity is obtained from the definition of \(C_W\) and the first equation, now that the lift \(C_{L,eq} = C_L(\alpha_{eq})\) is known:

\[
V_{eq}(t_t) = \sqrt{\frac{W}{\frac{1}{2}\rho A_{ref} C_{L,eq}}}
\]

(50)

Finally, knowing the velocity and incidence, the coefficients of drag \(C_{D,eq}\) and thrust \(C_{T,eq}\) can be calculated and the slope of the trajectory evaluated as:

\[
\beta_{eq}(t_t) = \frac{C_{T,eq} - C_{D,eq}}{C_{L,eq}}
\]

(51)

Note that if \(C_{T,eq} < C_{D,eq}\) the slope is negative and the trajectory is down. This is the case for a glider or when the engine is turned off since \(C_T = 0\). This simple system also shows that for a glider, if one neglects the change in Reynolds number hence in \(C_{D0}(Re)\), adding or subtraction mass at the center of gravity does not affect the equilibrium incidence \(\alpha_{eq}\) nor the slope angle \(\beta_{eq}\), and only affects the velocity on the trajectory which will change proportionally to \(\sqrt{W}\).

**Figure 7** Stable and unstable pitching moment curves (see online version for colours)

Considering the moment of the aerodynamic forces at the center of gravity eliminates the action of the weight. It is easy to show that, for static stability, one needs to satisfy the inequality:

\[
\frac{dC_{M,eq}(\alpha)}{d\alpha} < 0
\]

(52)
Indeed, if a perturbation, say a gust of wind, deviates the incidence from the equilibrium incidence by a $\Delta \alpha > 0$ or nose up, a negative pitching or nose down moment $\Delta C_{M,cg} < 0$ is needed to restore the equilibrium incidence, and vice-versa, Figure 7.

Figure 8 UC Davis entry at the 2006 Aero Design West competition (see online version for colours)

Another important point for the aerodynamic static stability of an airplane is the aerodynamic center (or neutral point) of the configuration. It is located between the aerodynamic center of the main wing and the aerodynamic center of the tail, proportionally to their areas and lift slopes as given by the linear model (neglecting the small fuselage contribution):

\[
\frac{x_{ac}}{l_{ref}} = \frac{A_m \frac{dC_{Lm}}{d\alpha}}{l_{ref}} x_{acm} + \frac{A_t \frac{dC_{Lt}}{d\alpha}}{l_{ref}} x_{act} + \frac{A_m \frac{dC_{Lm}}{d\alpha}}{A_t \frac{dC_{Lt}}{d\alpha}}
\]  

(53)

Note that its location is independent of $t_z$. It is defined as the point about which the moment of the aerodynamic forces is independent of incidence, i.e., it satisfies:

\[
\frac{dC_{M,0}}{d\alpha} + \frac{x_{ac} dC_{L}}{l_{ref}} d\alpha = 0
\]

(54)

Taking the derivative of the moment equation about the center of gravity, $C_{M,cg}(\alpha)$ and satisfying the static stability inequality reads:
Elimination of $dC_{M,0}/d\alpha$ between the two yields the following inequality for stability:

$$\left(\frac{x_{cg}}{l_{ref}} - \frac{x_{ac}}{l_{ref}}\right) \frac{dC_L}{d\alpha} < 0$$

(56)

Since the lift slope is positive, the condition reduces to:

$$\frac{x_{cg}}{l_{ref}} - \frac{x_{ac}}{l_{ref}} < 0$$

(57)

In other words, the center of gravity must be in front of the aerodynamic center. This is one of the most important design requirements. The static margin defines the distance $x_{a.c.} = x_{c.g.}$ as a percentage of the reference length as:

$$SM = 100 \left(\frac{x_{ac} - x_{cg}}{l_{ref}}\right)$$

(58)

7 Winglet design

Winglets are small aerodynamic surfaces, placed at the tip of wings, to improve their aerodynamic efficiency. Many different wingtip device geometries have been proposed, including raked tip, blended winglet, canted winglet, up/down winglet, spiroid, tip feathers, tip fence, etc., see Bertin and Cummings (2009). In a seminal 1921 paper ‘The minimum induced drag of aerofoil’, M.M. Munk, using inviscid flow and lifting line theory, showed that the most efficient winglets are placed at 90° from the main wing. Such winglets, in the upward position (placed on the suction side of the wing), have been studied by the author in a recent paper (Chattot, 2006), using inviscid flow including a viscous correction. The reader is referred to the paper for the theoretical details. An optimisation/analysis code has been developed to help students design winglets. For a given dimensionless winglet height $a_t$ (in reference to half span $b/2$) and target lift coefficient $C_{L, target}$, the optimisation code calculates the optimum distribution of circulation along the wing span and the winglet, as a generalisation of the lifting line theory to non-planar wings. The downwash is generalised as the normalwash, i.e., the component of the induced velocity normal to the dihedral shape of the wing. The 2D polars for the main wing and the winglet can be different, corresponding to different profiles, with high camber for the wing and lower or no camber for the winglet (the choice made here). Once the circulation is known, a design can be selected since, as in inviscid flow, there is an infinite number of possible geometries that will produce the given circulation. One of the simplest strategies is to achieve a constant local lift coefficient $C_l$ by designing the chord distribution according to the local circulation and the root chord or the local desired lift coefficient $C_l(\alpha_{opt})$ (corresponding to maximum $C_l/C_d$), as:
\[ c(s) = c(0) \frac{\Gamma(s)}{\Gamma(0)} \]  

or

\[ c(s) = \frac{2\Gamma(s)}{UC_l(\alpha_{opt})} \]  

\( s \) is the curvilinear abscissa. For the main wing, \( \alpha_{opt} \) represents the effective incidence, i.e., \( \alpha_{opt} = \alpha + \alpha_i \), where \( \alpha \) is the geometric incidence and \( \alpha_i \) the induced incidence. On the winglets, which are equipped with a symmetric profile, \( \alpha_{opt} = \beta = C_{l,\text{winglet}}/2\pi \), represents the toe-in angle, since the induced normal wash is zero. In Figure 9, a comparison of the optimum distributions of circulation and normal wash is presented for a simple wing and a wing with 20% winglets, for a given lift coefficient \( C_L = 2.0 \). The following comments can be made: the total lift is the same for both wings. The simple wing has an elliptic loading, with a high maximum root value, decreasing to zero at the tips, whereas the wing with winglets has a flatter distribution with lower root circulation and with higher loading at the winglet root. The downwash is constant for both wings, with a lower absolute value for the wing with winglets, which explains the lower induced drag given by:

\[
A_{ref}C_{Di} = -\frac{1}{U^2} \int_{\frac{S}{2}}^{\frac{S}{2}} \Gamma(s)q_n(s)ds
\]  

where \( S = (1 + a_t)b/2 \) represents the curvilinear length of the non-planar wing and \( q_n(s) \) is the normal wash such that \( q_n(s) = w(s) \) along the main wing, \( q_n(s) = v(s) \) on the left winglet and \( q_n(s) = -v(s) \) on the right winglet. The winglets carry a load which does not contribute to lift, since lift is given by:

\[
A_{ref}C_L = \frac{2}{U} \int_{\frac{S}{2}}^{\frac{S}{2}} \Gamma(s)\frac{dy}{ds} ds
\]  

where \( y(s) \) represents the dihedral shape of the non-planar wing, and \( dy/ds = 0 \) on the winglets. The winglets do not contribute to induced drag either since \( q_n(s) = 0 \) on the winglets. One should also note the flow singularity at the wing/winglet junction which shows as very large values of \( q_n \) and requires a fine mesh system to capture the solution (200 points are used in this calculation). The induced drag of the wing with winglet, \( C_{Di} = 0.0868 \), is compared to that of the simple wing, \( C_{Di} = 0.106 \), which provides an 18% decrease in induced drag and corresponds to an Oswald efficiency factor \( e = 1.22 \). The winglet geometry is shown in Figure 10, where the chord distribution is normalised with the winglet root chord \( c_{rw} \) and the winglet height with \( a_t b/2 \). The winglet corresponds to the area located below the curve, the root being the bottom edge of the box and the trailing edge the right edge of the box.
Figure 9 Circulation and normalwash distributions for wings with and without winglets

![Graph showing circulation and normalwash distributions for wings with and without winglets.](image)

Figure 10 Winglet geometry

![Graph showing winglet geometry.](image)

8 Trimming the glider for maximum distance

One of the glider design goal consists in achieving the largest distance on the ground, given an initial release point and altitude, assuming a perfectly quiet atmosphere. The situation is depicted in Figure 11(a). The glider is released from a height $H$ and will land at point at a distance $L$. The slope of the trajectory is $\beta$, hence the objective is to minimise $|\beta| = D/L$. With the simple equilibrium model, and the drag being given by:

$$ C_D = C_D^0 + \frac{C_L^2}{\pi eAR} $$

(63)

it is equivalent to minimise $D/L$ or $C_D/C_L$ as:

$$ f(C_L) = \frac{C_D}{C_L} = \frac{C_D^0}{C_L^0} + \frac{C_L}{\pi eAR}, \quad \Rightarrow f' = -\frac{C_D^0}{C_L^0} + \frac{1}{\pi eAR} = 0, $$

$$ \Rightarrow C_L = \sqrt{\frac{\pi eAR C_D^0}{C_L^0}} $$

(64)
so that the corresponding drag is $C_D = 2C_{D0}$. Geometrically, this corresponds to the point of contact of the tangent through the origin with the parabola, see Figure 11(b).

Figure 11 (a) Sketch of glider trajectory (b) optimum points on polar (not to scale) (see online version for colours)

9 Trimming the glider for maximum duration

Another strategy for a glider is to stay airborne the longest time, or maximum duration. This is achieved by minimising the speed of descend, i.e., for small angles, $|w| \approx V |\beta|$. As pointed out earlier, the equilibrium speed $V$ is proportional to the square root of the weight $\sqrt{W}$, but also to the inverse square root of the lift coefficient $1/\sqrt{C_L}$. Hence $|w| \approx C_D/C_L^{3/2}$. One obtains:

$$f(C_L) = \frac{C_{D0}}{C_L^{3/2}} + \frac{C_L^{1/2}}{\pi eAR}, \quad f' = -\frac{3}{2} \frac{C_{D0}}{C_L^{5/2}} + \frac{1}{2} \frac{C_L^{-1/2}}{\pi eAR} = 0,$$

and the corresponding value of drag is $C_D = 4C_{D0}$. This is represented by the square in Figure 11(b). Note that to change from maximum distance to maximum duration, the incidence is increased and the velocity decreased. Analysis of the results show that the ratio of the maximum duration time to the maximum distance time is $3^{3/4}/2 = 1.14$, a 14% increase in time. The distance achieved is however reduced, in comparison to the maximum distance calculated above, in the ratio $\sqrt{3}/2 = 0.87$, a 13% reduction.
10 Classical vs. canard configurations

A classical configuration consists of the main wing in the front and the tail in the back. In a ‘canard’ configuration, the two lifting elements are reversed (see for example the Wright Brothers’ flyer). The common wisdom is that in a classical configuration, the tail has a negative lift, whereas with a canard design, the forward control surfaces have positive lift. This makes the canard a desirable configuration for a heavy lift airplane. Many teams have used this design configuration, but only few have succeeded in having a stable airplane. See Figure 12 for illustration of the difference in cruise. In Figure 12(a) the moment of the main wing about the center of gravity is a nose down moment that needs to be balanced by a nose up moment from the tail. With the canard, Figure 12(b), the same situation requires a nose up moment from the forward lifting elements, which provide a positive lift.

Figure 12  (a) Classical vs. (b) canard configurations (see online version for colours)

However, it is possible to design a classical configuration in such a way that the tail will be lifting at take-off. The key point is to oversize the tail design, which is not a significant empty weight penalty, since it can be made out of light materials and it will lift more than its own weight. In this option, one requires that the center of gravity of the airplane be located aft of the main wing quarter-chord (main wing aerodynamic center). To do so, and still insure that the necessary static margin ($SM$) is preserved, the tail needs to be quite large, in order to move the aerodynamic center of the complete configuration sufficiently downstream. At take-off, when the airplane is fully loaded, the incidence is quite large, say $15^\circ$ or so, and as we know from thin airfoil theory, the main wing lift force will move close to the main wing quarter-chord. As the airplane rotates to take-off, the lift force moves towards the quarter-chord and passes in front of the center of gravity which creates a nose up moment that must be balanced by the positive lift force of the tail, see Figure 13(a). But in cruise, the main wing lift force moves...
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back past the center of gravity as the airplane speed has increased and the incidence decreased, requiring a negative lift force on the tail, Figure 13(b).

Figure 13  Classical configuration with lifting tail at (a) take-off and in (b) cruise (see online version for colours)

The equilibrium code is used to size the tail in order to achieve the above result. From the pilot point of view, flying such a configuration did not make any difference in handling qualities.

11 Conclusions

A simple approach to design a small remote control glider or airplane has been presented. It is based on a hierarchy of computer models, that help with the sizing of the wing and the tail, using a rapid prototyping code, estimating the take-off velocity with input and best estimate of rolling conditions, and finally, an equilibrium code that analyses the equilibrium in various phases of the flight, from take-off to cruise and descent and provides the flight envelop in which the airplane is controllable with the tail as a function of the static margin. The design of winglets that contribute to improved efficiency by decreasing the induced drag, is also discussed along with trimming the glider for maximum distance or maximum duration. Lastly, it is shown that with a classic configuration, the wing and tail sizing and the center of gravity location can be found to make the tail a lifting tail at take-off, which is desirable for a heavy lifter airplane.

References
