Validation of the Helicoidal Vortex Model using the S Sequence Data from the NREL’s Unsteady Aerodynamic Experiment

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Abstract

The helicoidal vortex model is validated using data from the S sequence of the NREL’s Unsteady Aerodynamic experiment. The steady analysis of the model ranges from 5 to 25 meters per second, while the unsteady process ranges from 5 to 15 meters per second with yaw angles as large as 45 degrees. The airfoil used in the analysis process is the S809, with a radius of 5.029 meters. It was found that not only does the model correlate extremely well with the global parameters of power and torque to the NREL database, but also does quite well for the local blade parameters. The local parameters of dynamic pressure, and the coefficients of normal and tangential force were examined at radial blade locations of 30, 47, 63, 80 and 95 percent. The best correlation for the model occurred before or up to the initial stall for a given radial location. It was found that all velocities and yaw angles were fully attached for conditions less than 8 m/s, and 20 degrees of yaw.
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Nomenclature

Adv - Advance Ratio
TSR - Tip Speed Ratio
$V_{\text{inf}}$ - Flow Velocity
RPM - Rotations per minute
R - Radius
pt - Power Coefficient
$\rho$ - Density
$C_l$ - Local Coefficient of lift
$C_{l,\text{max}}$ - Local Maximum Coefficient of Lift
$C_D$ - Coefficient of Drag
$C_N$ - Normal Force Coefficient
$C_T$ - Tangent Force Coefficient
$C_p$ - Coefficient of Pressure
$x_i$ - Normalized Chord position
$y_i$ - Normalized distance from axis of rotation
Re - Reynolds Number
V - Rotational velocity of Turbine Blade
$\nu$ - Kinematic viscosity
$c(r)$ - Local Chord length
$q(r)$ - Local Incoming flow velocity
betad - Flow Yaw Angle
tbd - Blade Setting Angle
skew0 - Artificial Dampening term
Cone Angle - Blade Flap Angle
I. Introduction

In the field of aerodynamic wind turbine design there is a conundrum that plagues the design process. How do you design a wind turbine accurately and cheaply? Even in this day of computing power this has not been done. The designers often have two choices: use a quick, computationally cheap method to analyze many possible designs, or use an expensive Navier Stokes, N-S, solver. While the first choice allows many turbines to be analyzed, the relative error in the solutions is too large to be of much use. On the other hand, the N-S solutions are difficult and extremely expensive to setup, limiting the designers to only a few possible system setups. Traditionally this means that a cheap, inaccurate method is used to narrow down the designs to a point where a N-S solver checks a couple of final design possibilities. Just imagine how the design process be improved if a more accurate two-dimensional analysis could be created, allowing far superior designs to be found before the Navier Stokes solution was applied.

One such model is the helicoidal vortex model, which can be used to predict the flow over the wind turbine blades. Not only is this method computationally cheaper than the N-S method, but it is more accurate than many other two dimensional solvers currently being used. In the following report the helicoidal vortex model will be validated under steady and unsteady flow conditions and compared to the National Renewable Energy Laboratory’s, NREL, Unsteady Aerodynamic Experiment or UAE. By comparing an exhaustive set of helicoidal vortex model data to the S sequence of data from the UAE experiments the model was validated. This validation includes steady and unsteady flow comparisons over a wide range of flow velocities and yaw angles, proving the models worth under a wide range of flow conditions.

During the validation process it was found that the helicoidal vortex model, does an excellent job in predicting the global flow parameters, and does a good job predicting the flow parameters over the blade. It was found that overall the system did very well until just after stall was reached at a given radial location. Stall was found to initially occur at a velocity of 8 m/s and 30 degrees of yaw at around 35 percent of the chord. As the speed increased larger sections of the span became stalled, until almost the entire blade had reached stall at 15 m/s. The global values of power and torque had excellent correlation with the experimental data until after ten meters per second, and yaw angles up to 20 degrees, even when a significant portion of the blade became stalled in the flow. Dynamic pressure was one feature that always performed very well in the analysis process associating almost perfectly with the experimental values. The coefficients of normal and tangential force were also found to have excellent correlation for all blade locations and yaw conditions for velocities less than ten meters per second. The best correlation occurred when the local point of analysis reached the point of maximum power output at $C_{l,max}$.

1. Background

The helicoidal vortex model was originally developed to design a rotor that will produce the maximum torque for a given thrust and tip speed ratio. This model is based upon the Goldstein model, and allows interactions to take place between the blade elements. To construct the vortical structure behind the rotor the power produced by the rotor matched with the power absorbed by the rotor according to actuator disk theory, which is related to the axial flow velocity. Two dimensional viscous polar data from Xfoil is also used to increase the accuracy of the solutions for power and torque produced by the blades. In order to allow a solution to be found if the angle of attack is greater than the angle for $C_{l,max}$ an artificial viscosity term is added.
into the system [1]. In order to analyze a wind turbine under steady and unsteady conditions two codes developed by Professor Jean-Jacques Chattot are used. The steady code, turbana.f analyzes flow with no yaw, while the turbuns.f code analyzes flows with yaw components. When used together these two codes provide excellent matching to experimental data. One definite advantage to this analysis process is that the helicoidal vortex model realistically holds onto the vortex sheet and allows it to extend out into the far field, as it does in reality. This is in direct contrast with a Navier Stokes solver where the artificial viscosity causes the vortex sheet to break down after one rotation of the blade.

The Unsteady Aerodynamic Experiment was conducted at the NASA Ames Research Center at Moffett Field, California, in the 80 x 120 ft wind tunnel. The purpose of the experiment was to gain quantitative full scale data for a real wind turbine in an anomaly free environment [2]. By creating a database of aerodynamic and structural measurements for a full sized wind turbine a data set was created with which computer analytical models could compare to validate their accuracy. The test turbine was a modified Grumman Wind Stream-33. The turbine blade used was a S809, with a 5.029 meter radius, and twist as identified in the Unsteady Aerodynamics Experiment Phase VI paper by M.M Hand and colleagues. The blade profile for this experiment begins at 25 percent of the radius. The analytical profile for the S809 was taken from the NASG website and contains 62 data points defining its profile. The experiment was performed for velocities between 5 and 25 meters per second, and for yaw angles from zero to one hundred eighty degrees under the S sequence setup. For the S sequence the turbine blades were upwind of the tower, with a zero degree cone angle. The blade pitch was set at 3 degrees, and the rotation rate was set at 72 RPM. Pressure taps were located at 22 points, as seen in figure I.1, along the chord of the airfoil at 5 radial locations. These locations were 30, 47, 63, and 95 percent of the span.

![Figure I.1: Pressure Tap Locations](image)

In order to validate the helicoidal vortex model, an exhaustive set of runs was performed on the steady and unsteady conditions, as seen in table I.1. This exhaustive set of runs was performed in order to accurately portray how changes in the velocity and yaw angle, either small or large affect the solutions. By comparing the analytical and experimental solutions the effects of alterations in velocity, yaw angle and radial location along the blade could be examined. Also the affects could be studied to analyze how the parameters of dynamic pressure, and the coefficients of normal and tangential force become altered as well. The global parameters of power generation and torque were also compared, as ultimately these are the desired properties of a wind turbine.
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Table I.1: Flow Velocities and Yaw angles Analyzed at 30, 47, 63, 80, and 95 % of the span

II. Steady Analysis

1. Polar Setup
   For turbana.f and turbuns.f a polar input file is required to run the simulation. The codes can input from one to many polar files depending on whether the user requires the polar itself to change over the span of the blade. In order to create a polar file the user must run Xfoil for the given blade from an angle of attack of zero, or lower, until Xfoil will no longer converge. After creating the polar file, it requires some editing. Using either notepad or wordpad on a windows machine the polar file can be opened. In the upper left hand corner of the code place a value of one. This tells turbana or turbuns that this is the first polar file in the series. If only 1 polar file is to be used a value of 1.1 should be placed below the final alpha value. This tells the code that no other polar files need to be read. After these formatting issues have been dealt with, the data must be examined. In order to ensure a parabolic curve fit at every point along the polar file some interpolation is required around the local maximum and minimum values. Using a linear interpolation a point should be added before and after every local maxima or minima. This interpolation should create values for alpha, C_l, and C_d. At the end of each interpolated line it would be advised to label the line interpolated for future reference. Once this has been completed the file can be saved and is ready for use in the turbana or turbuns codes.

2. Steady Code turbana Run Procedure
   In order to run the steady code the following steps should be taken. The data file turbana.dat must be modified to the given flow parameters. When comparing to the NREL
database for sequence S the rotor radius (R) of 5.0386, max chord length of .2, and root location of .25 do not require alteration. The parameters that must be modified are the wind velocity (V\textsubscript{inf}), blade setting angle (tbd), air density (\(\rho\)), advance power ratio (adv) and the power coefficient (pt). The first three values can be found by finding the average of the NREL cycle data that is provided or by running the sorting code, which uses the raw data to find the average values. The advance power ratio requires a simple calculation to be found.

\[
adv = \frac{1}{TSR} \frac{V\text{BladeSpeed}}{V\text{TSRadv}} = \frac{60 * V\text{inf}}{2 * \pi * RPM * R}
\]  

(1)

The power coefficient is found through an iterative process. First an initial guess for pt is placed into the input file. This guess is made utilizing the fact that the power coefficient must be greater than a minimum value, which is defined by the following:

\[
pt > -\frac{16\pi(adv)^3}{27}
\]  

(2)

![Image](https://example.com/image.png)

Figure II.1: Example data input file for turbana.f

Now that all of the turbana.data inputs have been placed into the turbana.dat input file the code, as seen in figure II.1, can be run. After compiling and executing the program, the code will pause before reading and displaying the polar file, simply hit enter to continue. The code will then ask if you wish to read a file. Press one for yes or zero for no and press enter. The code will then ask if you wish to run it as a viscous or inviscid case. Unless required to force the solution to converge the code should always be run for the viscous case. The code will then ask for the number of iteration steps that you wish to run. You want to run the code until the solution has converged, meaning the term dgx is on the order of 10e-8. One thousand iterations is usually sufficient to reach this order of accuracy. Once this has occurred input zero as the required iterations, and the output files will be created. Also outputted to the screen is the new power coefficient. If this value differs from that in your input data file, replace the old value with the new one in turbana.dat. Then copy turbana.out into turbana.in, and restart the code, this time loading the previous data when prompted. Once the power coefficient term stops changing, a steady solution has been found.

To summarize

1. Place correct turbine data and power coefficient guess into data file.
2. Run turbana.f code
3. Viscous run so choose 1
4. Run iterations until dgx is on the order of 10e-8, (1000 iterations usually)
5. Read new pt value and place into data file.
6. Copy turbana.out to turbana.in
7. Rerun code, this time loading data when prompted
8. Stop rerunning when pt no longer changes

3. Steady Analysis Introduction
A steady code analysis is important for a few different reasons. First the power
coefficient found from the steady code is required as an input for the unsteady turbine analysis.
Another important feature of the steady code analysis is that the power output of the steady code
can be compared to the actual NREL data for the identical situations. Therefore this system can
serve as a baseline to see how well the code is solving the flow equations.

4. Flow Separation
One of the most important initial steps during the steady analysis was to determine when
the flow was fully attached to the airfoil, as the theory and accuracy of the code will be greatest
when this is true. Once the flow separates from the airfoil, the theory will no longer be valid, and
a more exact solver that examines three-dimensional effects will be required. In order to
determine the flow attachment the $C_l$'s for a given point on the airfoil can be compared to the
polar of that particular airfoil. For the S809 it was determined that flow separation occurs when
the point on the airfoil is greater than the 29$^{th}$ point on the polar for a Reynolds number of
500,000 from Xfoil. For ease in examination the points on the airfoil output their location on the
polar in terms of the points that define the S809 polar. Therefore if any point has an output
greater than 29 then the flow is separated at that point. What was found was that the flow is fully
attached until 9 meters per second. This means that for zero yaw input the flow is attached until
this point. Therefore the code will be the most accurate for flow velocities less than 9 meters per
second. This was graphically represented in figure A1.

5. Blade Stall
Blade stall is one feature that is very important to examine when designing a wind
turbine. An airfoil is defined as being stalled when a is greater than the $a$ of $C_{l_{\text{max}}}$. This occurs
when the index of a point on the polar is greater than a value of 39, the location of the maximum
lift for a Reynolds number of 500,000. One of the most important features of stall is past the
blade stall point increases in alpha will not only decrease the lift but increase the drag as well.
For a wind turbine this translates into less torque on the blade due to drag losses and therefore
less power generation. In order to identify where the flow is stalling over the blade, the points
corresponding to the input 2-d polar file are outputted to the screen. The spanwise locations of
the points are placed using a cosine distribution from .25 to the full radius of the blade. Given the
polar point at each location the flow can be identified as pre or post stall.
Using this data figure II.2 was created. In figure II.2 each data point represents a blade position where the flow is stalled at that particular velocity. One interesting feature that is clear is that according to this analysis there is no stall below 10 meters per second. This seems to correspond with the power curves, which show a local maximum power output at approximately 9 meters per second. The stall of the blade increases relatively rapidly after the initial stalling of 35 to 50 percent of the span. From 10 to 12 meters per second the stalled region increases from around one quarter of the blade to over half of the blade span. In comparison from 14 to 25 meters per second only a ten percent section of the blade becomes stalled. Another interesting feature is that the blade separates in the increasing radii direction much quicker than it moves in the decreasing direction.

6. Power Output

The power output as stated previously for the wind turbine can be found using the power coefficient term, pt. The equation for the power with pt is:

\[ Power = 0.5 \rho \Omega^3 R^5 pt \]  

where \( \Omega = \frac{2\pi RPM}{60} \), \( \rho \) is the density, \( R \) is the blade radius, and RPM is the rotations per minute. From this analysis a figure can be made that relates the steady code power output to the equivalent NREL Unsteady Aerodynamic Experiment, UAE, data. This curve can be seen in figure II.3.
From this figure some very important information can be extracted. For the velocity range of 5 to 10 meters per second the power differs only slightly between the NREL data and the analytical code output. After approximately 9 meters per second the flow has begun to separate from the airfoil so some drift in the power output should be expected and is seen. From figures A1 and A.2 it was seen that the flow at 9 meters per second has a section where the flow is separated, but not yet stalled. This could explain the hump that is seen in the power curve at 9 meters per second. Due to the fact that the flow is separated, but not stalled, it means that a large section of the blade is sitting very near or at the $C_{l,\text{max}}$ value. Therefore the blade is running at its most optimal condition at 9 meters per second, creating the maximum power output under steady conditions. The smooth nature of the analytical output is a function of its dependence on only the two dimensional effects of the airfoil. After the final point on the two dimensional polar is found from Xfoil, a final point at zero is extrapolated so the code has some nature to base itself on for large angle of attack. This extrapolation can be seen in figure II.4. From this figure the assumption of a linear decrease of $C_l$ is represented visually. This assumption while not perfect provides a reasonable assumption as to the differences between the analytical and experimental solution. It is possible that this linear extrapolation may not be physical. It is likely that the $C_l$ has a rapid drop, which could explain the plateau in the NREL power curve. Since the dynamic pressure increases, but the lift of each section decreases, the result is a leveling of the power curve that will not be found in the analytical solution. The experimental data also shows that the power output increases after 20 meters per second. It is likely that the rapid drop in $C_l$ has already occurred by this velocity range. So the increase in the dynamic pressure when coupled with a no longer rapidly decreasing $C_l$ creates an increase in power output. This is a byproduct of using a two dimensional analysis like Xfoil, which does not do a great job with analyzing flow after stall has occurred.
7. Torque

Torque is also a vital output for the design of wind turbines. The torque created on the blades translates directly into the power that can be generated by the turbine. The relationship between the torque and output power is:

\[ \text{Power} = \text{Torque} \times \Omega \quad (4) \]

where the torque is in Newton meters. Due to this relationship the curves of power versus velocity, and torque versus velocity will have the identical shape and will differ only in their magnitudes. It is important to realize that this relationship will display an aerodynamic power, not the actual power, as there are mechanical losses that will cause the system to lose some energy. Often the torque will actually be a more desirable output than the power as it can be compared directly to the output of the normal force and tangential force through the following equation

\[ C_{TQ} = (C_N \sin (\phi + \beta) + C_T \cos (\phi + \beta)) \cos (\text{cone}) \quad (5) \]

where \( \phi \) is the local twist angle, and \( \beta \) is the blade pitch angle. The cone refers to the value of the flap angle of the blade, which is zero for the S sequence of analysis. This value is included to maintain the thrust convention positive perpendicular to the plane of rotation. The average torque versus velocity can be found in figure II.5.
Other important outputs include, but are not limited to the normal and tangential force coefficients, and the dynamic pressure often referred to as Qnorm. These coefficients are very useful as they are dependent on the lift and drag of the wind turbine, but also provide meaningful data that can be found through experimental methods. In the following sections the coefficients of normal force and tangent force will be discussed.

8. $C_N$ and $C_T$

The normal force coefficient and tangential force coefficient are defined as shown in figure II.6. The normal force is defined as the force perpendicular to the chord line of the blade. The tangential force is defined parallel to the chord line, in the negative $x$ direction, where $x$ increases from the leading to trailing edge. Not only can the normal and tangential forces be found, but by relating the angle of attack to the tangential and normal forces, one can switch to the lift or drag of the blade, or vice versa. By integrating the coefficient of pressure over the change in $x$ the coefficient of normal force is found.

---

**Figure II.6: Aerodynamic Force Coefficient Conventions [2]**
Similarly the coefficient of tangential force is found by integrating the coefficient of pressure over the changes in y. This corresponds to the analytical expressions seen in equations 6 and 7

$$C_N = \sum_{i=1}^{\text{# of taps}} \left( \frac{C_{p_i} - C_{p_{i+1}}}{2} \right) (x_{i+1} - x_i)$$  \hspace{1cm} (6)

$$C_T = \sum_{i=1}^{\text{# of taps}} \left( \frac{C_{p_i} - C_{p_{i+1}}}{2} \right) (y_{i+1} - y_i)$$  \hspace{1cm} (7)

In this way the $C_N$ and $C_T$ analytical and experimental solutions can be compared.

In figures A4 through A8 the $C_N$ curves versus velocity can be found. What can be seen in these figures is that at all spanwise locations the solutions are not unreasonable for values up to approximately 10 meters per second. After this point the two-dimensional polar data becomes questionable as the flow is no longer attached to the blades. This error is visually apparent, as the flow solutions are not very good for velocities greater than 10 meters per second. It does appear relevant to note that the while the analytical solution does not have the exact values the trends that the solution takes closely mirror that of the experiment. The largest flow errors occur at 95% and 30% of the span, positions where the largest three-dimensional effects would be most important. The solutions at the three other radial locations are far better. In fact the solutions at 63 and 80 percent of the span are reasonable up to velocities of 14 meters per second.

The $C_T$ curves as seen in figures A9 – A13 do a good job predicting the tangential force up to a velocity of 9 meters per second. It is at this point that a drastic difference occurs between the experimental data and the analytical solution. Between 9 meters per second and 15 meters per second, depending on the spanwise location the tangential force drops quickly and significantly. At 95 percent of the span the solution is fairly accurate until 14 m/s, at 80 % until 13 m/s and so on. It would appear that the stall velocities at each of this locations could be causing the problem. Looking at figure A2, the initial location of stall at a velocity corresponds fairly well to the sharp changes in tangential force. This would seem to imply that the polar file from Xfoil does a fairly good job approximating the flow up until stall, but beyond this point better polar data is required. This would also explain why the 30 % and 95% span curves do not have a steep drop, but a slower smooth one, as they both stall at much higher velocities than the other locations. The slow, delayed analytical solution drop could be function of the additional data points that were added to the polar file. When the polar file was created a linear extrapolation was introduced that placed a final data point at 90 degrees where the lift was zero, and the drag had a value of 2.

9. Dynamic Pressure

The dynamic pressure of a system is very important as it is used to normalize the pressure into $C_p$, which all other aerodynamic components are calculated from. The Qnorm value, as referred to in the NREL database, is the stagnation point dynamic pressure that is corrected for the centrifugal force, and has the same units as pressure, usually Pascals. The dynamic pressure is an excellent check to see whether there is an inherent error in the analytical process, because if this value has large errors so will the rest of the aerodynamic components.

The dynamic pressure is a flow field characteristic and therefore should be very accurate under steady flow conditions. In figures A14 –A18 this was found to be true. No matter what the velocity, or radial location the analytical dynamic pressure value almost perfectly matched the experimental values. The main differences are that the analytical results are perfectly smooth,
while the experimental results are not. This could be a factor of error in the experimental dynamic pressure value. In order to solve experimentally for the dynamic pressure value the tap with the maximum pressure was assumed to be the stagnation point. Yet this might not have been perfectly true as the taps do include some error, but also do not cover the airfoil perfectly. It is possible that the taps were very close to, but not exactly at the stagnation point causing some small differences in values.

The disparity in the curves is almost negligible except at 95% of the span where the three dimensional and rotational effects are most significant. Not only is the blade moving fastest at this point, but the tip vortices could also be causing small alterations in the dynamic pressure. Yet even with these possible errors, the analytical and experimental dynamic pressures are close enough over a wide range of velocities to ignore any discrepancies.

10. Effect of Reynolds Number

The effect of Reynolds number must always be taken into account when analyzing any airfoil. One key note to this analysis is that only one polar file is being used from the entire blade radius, and all the velocities. While it is understandable that the blade radius could be approximated to have a single Reynolds number, it is unclear what Reynolds number would be a good approximation for the system. Recall that \( \text{Re} = \frac{q(r)c(r)}{v} \), where \( q(r) \) is the incoming velocity to the wind turbine. Therefore the Reynolds number range is from approximately four hundred thousand to one million. In order to examine how the Reynolds number affects the outputs of the system, polars were created for the Reynolds numbers of 4e5, 5e5 and 7.5 e5. The polar files for the varying Reynolds numbers are displayed graphically in figures II.7 and II.8.

![Polar Comparision at Varying Reynolds Numbers](image)

Figure II.7: Polar Comparison for Varying Reynolds Numbers

In figure II.7 the entire polar range can be seen from angles of attack of zero to 90 degrees. As previously explained the linear region of the polar after a \( C_D \) of .2 is due to the linear extrapolation to the flat plate drag of 2 at the angle of attack of 90 degrees. The first note that can be made is that the differences between the four and five hundred thousand are larger than the differences from 500 to 750 thousand for large values of drag. This would imply that for high \( C_D \) areas an assumption of 5e5 would not create significant errors. From figure II.7 the most
interesting region, with $C_D$ less than .2, is not clearly visible so the analysis must be shifted to figure II.8.

In figure II.8, the differences between the individual curves can be seen. The first initial difference that can be found is that in the initial linear region where the $C_l$ is increasing with minimal increases in drag the curves are very similar. They all contain similar slopes and values. As expected the highest Reynolds number polar file has a slightly better lift to drag ratio than the others. One main difference occurs around a $C_D$ of 0.06. The highest Reynolds number curve reaches a larger $C_{l,max}$ at a lower drag value than the other two curves. The large Reynolds number then makes the airfoil more effective, as it increases the lift over all angles of attack and decreases the drag at $C_{l,max}$. These are the basic differences between the polars with varying Reynolds numbers.

![Polar Comparison at Differing Reynolds Numbers](image)

Figure II.8: Increased view of Reynolds Number Comparison

In order to understand how these differences would alter the flow solutions the polar files were run under the same steady flow conditions and their outputs were compared. These solutions were plotted on figures A19 through A21.

What was found from figure 19 is that the Reynolds number does not have a large effect on the power output before separation occurs at 9 meters per second. In fact it is at approximately 9 meters per seconds that the first discernable differences between all three Reynolds number results can be seen. As one would expect the power output increases with the Reynolds number after 9 meters per second. The five hundred thousand value seems for power to be the average curve between the high and low Reynolds number effects as it keeps the shape and is more accurate than the four hundred thousand value for velocities less than 9 meters per second.

The CN at 63% curve as well as the CT curve were also used to analyze the effects of Reynolds number. For both of these curves the changes due to Reynolds number for velocities less than 10 meters per second are very small, meaning that even a large change does not significantly affect the system. Once again the middle value of 5e5 seems to provide the best fit for the data as it best minimizes the average error for the low end velocities. One interesting feature for the Ct curve at 4e5 is that the slope after 10 meters per second is very nonlinear,
unlike the other curves. This is due to the curve fit of the plotting program, as the value of the curve for 4e5 is much larger than the other comparable values.

**Tap Number Effects**

When analyzing aerodynamic coefficients it is always important to examine where and how the data points differ. The major difference between the analytic solution and experimental results are the number of data points examined along the airfoil. For the NREL UAE experiment there are 22 data points spread across the airfoil. The S809 profile from which Xfoil finds its polar values contains 61 data points spread with a cosine distribution. In order to examine the differences that arise from the disparity in data points, the coefficients of lift and drag were calculated. This was done using the Xfoil parameters of chord location, relative thickness, and the coefficients of pressure and friction. These values were then compared to the Xfoil outputs of lift and drag for the same angles of attack. In order to solve for the coefficients of lift and drag of the airfoil the $C_N$ and $C_a$ values were calculated using outputs from Xfoil. The equations for the normal force coefficient, $C_N$, and the axial force coefficient, $C_a$ were found using equations 8 and 9.

\[
C_N = \int_{LE}^{TE} (C_p\,dx) - \int_{LE}^{TE} (C_p\,dx) + \int_{LE}^{TE} (C_f\,dy) + \int_{LE}^{TE} (C_f\,dy)
\] (8)

\[
C_a = \int_{LE}^{TE} (C_p\,dy) - \int_{LE}^{TE} (C_p\,dy) + \int_{LE}^{TE} (C_f\,dx) + \int_{LE}^{TE} (C_f\,dx)
\] (9)

The integrals are from the leading edge, LE, of the airfoil to the trailing edge, TE, and the subscript u refers to the upper surface, while the subscript l refers to the lower surface. These two terms can then be combined with the angle of attack to find the lift and drag of the airfoil. The coefficients are defined as follows.

\[
C_l = -C_a \sin(\alpha) + C_N \cos(\alpha)
\] (10)

\[
C_d = C_N \sin(\alpha) + C_a \cos(\alpha)
\] (11)

This analysis was first performed on a NACA 0012, from Xfoil, which has 140 data points defining the profile of the airfoil in order to validate the equations. The results can be found in figures II.9 and II.10.

![Figure II.9: NACA 0012 Data Point Analysis Comparison of Lift](image)
What is clear from this analysis is that the airfoil can be very sensitive when it comes to the number of points in the airfoil profile. The reason why the Xfoil output differs from the 140 data point set is that the trapezoidal integration used to integrate equations 8 and 9 is not as accurate as the Xfoil integration. Also Xfoil will internally interpolate extra points to calculate during the analysis phase. Yet this difference is only apparent in the drag calculations, as it is more sensitive due to its small magnitude.

Now that the analysis has been proven the same type can be applied to the S809 to find the effect of the small number of taps along the airfoil. What quickly becomes apparent is that once again the change in the number of data points does not significantly affect the coefficient of lift, as all the curves are very close to each other, as seen in figure II.11.

The drag is a different story. Due to its small magnitude the number of analysis points has a greater effect on the coefficient of drag. From figure II.12 it is clear that the number of points has significantly altered the value of the drag from 28 percent difference at 0 degrees angle of attack to 54 percent error at 15 degrees angle of attack. This error is small, but could add up to cause some discrepancies between the experimental and analytical values.
The next step in this process is to examine how these differences in the coefficients of lift and drag affect the system outputs like the coefficients of normal and tangential force. If a large discrepancy results from the number of taps then an inherit error must be taken into account during the analysis phase. In order to do this one needs to only refer to figure II.5. From this figure it becomes clear that the tangential and normal force coefficients are:

\[ C_N = C_l \cos(\alpha) + C_d \sin(\alpha) \]  
\[ C_T = C_l \sin(\alpha) - C_d \cos(\alpha) \]

Utilizing equations 12 and 13, the \( C_N \) and \( C_T \) values were found over the previous range of angles of attack. As expected the accuracy of coefficient of normal force as a function of alpha was very good, as seen in figure II.13. This would seem to imply that very little error is introduced into the system from the error in the drag coefficient when it comes to the normal force. In figure II.14 the \( C_T \) versus alpha can be found. In this figure once again the number of data points had little effect on the tangential force coefficient. This is a very important result. By comparing the Xfoil output for a range of data point vector sizes it was determined that very little error is introduced to the NREL data from the use of only 22 taps.
Another step to ensure the accuracy of the tap pressure integration was to plot the coefficients of pressure as functions of the normalized chord length and thickness for the given alpha values. The purpose of this was to examine how the number, and location of data points will alter the coefficient of pressure and therefore all the aerodynamic coefficients. These $C_T$ versus normalized chord thickness can be found in appendix A figures A22 through A25. Despite the small number of tap locations along the blade the NREL did an excellent job fitting to the data over all angles of attack. The error that did occur was most prevalent at the lower angles of attack, and that was a difference in the slope of the lower surface curves at around one half of the chord length. In figures A26 to A28 the coefficient of pressure was plotted versus the normalized thickness of the airfoil. Like in the previous set of figures the matching of the NREL tap locations to the Xfoil data points was excellent. The error that did occur was on the upper surface near a thickness of zero, and on the lower surface at a value of negative .75. These differences appear to be equivalent to the errors seen previously, but are more easily seen due to the small magnitude of the figure.

### III. Unsteady Flow

The helicoidal vortex model is used to analyze the unsteady flow into a wind turbine using the code turbuns.f. This code is very similar in running setup to its steady counterpart, turbana.f, which was previously discussed. Like turbana.f, the unsteady code can have multiple polar file inputs and also contains a very similar input file. In the following sections the setup and running of this code will be discussed.

1. **Data File Setup**

   The first step in the process for the turbuns.f code is to run the steady case for the same velocity using turbana.f. This will do a few things for the user. First it will ensure that the polar file for the system has been properly setup and edited as discussed in the steady portion of this report. Secondly, this will provide the base pt value that needs to be inputted into the turbuns.dat file. Lastly, if the same mesh and vortex sheet geometry are used the turbana.f output file can be inputted into the turbuns.in input file to increase the convergence speed of the initial run. In order to edit the turbuns.dat file using a windows machine, open the file using either notepad or wordpad. The first noticeable difference is the increase of information contained in the turbuns
data file from the steady data file. Many of these parameters will not require alteration, for a desired turbine setup.

A typical data file is displayed in figure III.1. The discretization of the points on the blade, jx, is the first term in the file. This number must be less than 102 to work, and the points will be distributed using a cosine distribution. The dx0, and xstr values will not have to be changed under most, if any running conditions, unless the convergence of the code itself requires them to be altered. The xtrefftz value refers to the distance of the far field location, which is often placed at 20 times the blade radius, and nsec is the number of sectors. The last value of the section that will remain unchanged under most systems is the ndelta parameter. This value refers to the number of steps that the solution will be found per two pi, or 360 degrees. The default value of 24 informs the code to move 15 degrees with every time step. The next value is the skew0 parameter. The skew0 value is a weighting parameter that artificially assists the system to converge. The effects of this value will be discussed later, but it must be gradually decreased from a value of one to a value of 0.5, or as close to 0.5 as possible maintaining stability. When the code is run for the first time under given conditions the skew0 value should be set to 1. Unlike in the steady code the viscous, inviscid switch is located in the data file. This is because most cases are run with viscous effects. The iteration bound, relaxation factor and artificial viscosity for separated flow, and dynamic viscosity do not require alterations.

The final sets of data inside the file are dependent on either the wind turbine, or the specific data set being run. The root location of .25, max chord of .2, and rotor blade radius 5.026 meters of are functions of the data sequence and do not alter for the entire S sequence. The final values are dependent on the particular system setup. The advance ratio (adv) can be calculated as before using equation 1 from the steady analysis. The power coefficient (pt), is the value found from the steady analysis of the equivalent steady setup. This provides a base setup for the code to work from, but there is no need to iterate on the power coefficient in this code. The blade setting angle (tbd), air density (ρ), flow velocity (V_{inf}) and flow yaw angle (betad) can be found from either the average NREL S sequence data or can have the averages computed from the raw data using the sorting code. The final value that requires some explanation is rloc, the blade output location. The blade output location is the normalized spanwise location at which the local properties, i.e. C_T or C_N, will be calculated per run. Therefore this value will always be greater
than a quarter and less than one. To match the NREL database use the chord locations of .3, .47, .63, .8 and .95. These five locations will be discussed at length later in the report.

2. Unsteady Code Run Procedure

Now that the polar and data files have been prepared the code is ready to be executed. After compiling and initiating the program, the code will pause before reading and displaying the polar file. Like before hit a key to continue. The code will then prompt the user to press one if an input file is to be used or 0 if not. For the initial run with a skew0 value of one, the user should press zero if no input file is available, and hit enter. If the input file from turbana.f is to be used then the user should press one and then enter. The code will then ask for the number of time steps that you wish to run. In order to run the code to steady state the solution must run to a distance of at least 20 full rotations to reach steady state, or at least 20 times the ndelta parameter. For an ndelta of 24 this means at least 480 time steps. It also is important to run the code for an integer number of rotations, as every time the code restarts from an input file the blade-starting angle resets to zero. After the code has finished running for the given time steps it will ask if you wish to run for more. Simply input zero and hit enter. The code will then make the output files.

Now that the solution for a given blade section location and skew0 parameter have been found some more runs are required. The data file should be opened and the skew parameter should be decreased. By decreasing the skew0 term, the artificial dampening inside the system will be reduced. The turbuns.out file should then be copied to the turbuns.in file for the next run. The entire code can be run again, except this time a file should be inputted when prompted. The typical decrease in the skew0 parameters that works well is to use an initial value of 1, then .7, .65, .6, and then .55. It is unlikely that the solution will converge at a skew0 value of .5. Now that the solution has been converged for a given blade section location the code must be run for all the other desired blade locations. Fear not, the code and ramping of the skew0 value does not need to be repeated. Instead simply load up the latest output file, and change the rloc parameter to the desired value in the data file and run the code. The difference here is that the input file is not dependent on the rloc value, but rather contains information for the entire blade. Therefore the converged input file will work for all the blade locations.

3. File Output Summary

There are many different parameters that could be required from the analysis of a wind turbine. These can range from the power output to the lift distribution over the blade. Due to this fact a variety of output files are created. The following list contains the system output files, and components as they are outputted.

- turbuns.xyz – tip vortex x,y,z coordinates
- turbuns.rg – radius (r), circulation Γ(r)
- turbuns.rw– radius (r), downwash w(r)
- turbuns.ru - radius (r), axial interference u(r)
- turbuns.rv - radius (r), radial interference v(r)
- turbuns.rcl - radius (r), local coefficient of lift c_l(r)
- turbuns.rat - radius (r), angle of attack at(r)
- turbuns.rcp - radius (r), xle(r), xte(r), t(r), phi(r)
- turbuns.cdcl – c_d(r), c_l(r) polar
turbuns.rre – r, Local Reynolds Number rey(r)
turbuns.acde – adv, c_o/π, eta numerical
turbuns.all – summary file
turbuns.out – output file for restart
turbuns.xgi – x(i), g(i, jloc,1)
turbuns.ncp – Total rotational angle n(t), Cp(t)
turbuns.nqnt - Total rotational angle n(t), Qnorm(t), C_N(t), C_T(t)
turbuns.ng1 - Total rotational angle n(t), g1(t)
turbuns.nt - Total rotational angle n(t), Torque tau(t)

4. Run Procedure Summary
   1. Input proper data into turbuns.dat with a skew0=1.0
   2. Run turbuns.f
   3. Press zero for no input file, or 1 if input from steady case is to be used
   4. Run for at least 20 times ndelta (i.e. 480 steps)
   5. Copy turbuns.out to turbuns.in
   6. Decrease skew0 parameter
   7. Rerun code until lowest possible skew0 has been achieved.
   8. After convergence alter rloc value and rerun with lowest converging skew value.

5. Effects of Artificial Dampening, skew0
   The artificial dampening coefficient, skew0 is used to assist in the convergence of the
   solution during the time steps. This is required to enforce stability onto the system and prevent it
   from becoming unstable, stopping the code from running. The minimal skew0 value, where the
   code is no longer assisting in the convergence of the code is one half. In order to ensure a
   converged solution a few steps must be taken. The skew value must be set rather high, and then
   ramped down towards a minimal value where the solution will occur, without diverging. If this
   process is not followed then the solution will not only fail to converge, but in most cases will
   diverge. Since the code is unable to converge in the proper number of time steps the solutions are
   incorrect due to the instability and divergence of an improperly setup solution. This can be seen
   visually in figure III.2.

![Image](https://via.placeholder.com/150)

Figure III.2: Local $C_L$ versus Radius location for independent Skew0 values at 5 m/s, 5 deg Yaw

In this figure III.2 different skew values were run each independently of each other. It is quickly
noticed that the solutions not only significantly differ, but are not even converging towards a
similar answer. This instability and inaccuracy is the reason for the artificial dampening factor.
By not ramping down the artificial dampening the solutions are not converging, creating the solutions that are seen.

This inaccuracy is easily corrected. By running the skew0 dampening factor at high values initially, and then ramping down the system a stable accurate system is created. It was found that a systematic approach worked best at this and so skew0 values of 1, 0.7, 0.65, 0.6 were used. It was found that for values of 0.55 the solution was only barely stable enough to run, but was actually beginning to diverge at about three quarters through the time steps. The solution was never able to converge at a skew0 value of one half. Therefore by following these simple steps a converged natural solution was found.

In figures III.3 and III.4, the skew0 variable was ramped down slowly. Initially the code was run to a steady state, 20 full revolutions, for a skew0 value of 1. The artificial dampening was then decreased to .7, and the input file was loaded in. The system was then run until steady state again, and the process repeated itself. The skew0 values of .7, .65, and .6 were plotted for 5 meters per second at five degrees of yaw. From the full view position no difference can be seen. By looking to figure III.4 the differences between the three curves can be seen. The difference is small, but it means that the solution is remaining stable and is moving towards a periodic state solution that would occur if the dampening ratio were not required. The small alterations also mean that the initial solution from a dampening of 1 was not very far off, so the need for the decrease is not required for the solution, but rather to make the analytical solution match the theory. This is in vast difference to figure III.2 where the different dampening ratios caused very different coefficient of lift curves.

It was found that even at the mild unsteady condition of 5 meters per second and 5 degrees of yaw that the code would not converge with a dampening ratio of 0.5. The solution would converge with a value of 0.55, but as stated before, the solution was no longer stable, and was barely able to run for 20 revolutions without diverging completely. This difference can be seen in figure III.5, where all the skew parameters are plotted. Due to this result it was found that an artificial dampening ratio of 0.6 worked best for all cases without a diverging solution.
6. Unsteady Stall Analysis

Due to the helicoidal vortex model used to solve the flow over the given conditions a stall analysis is required. After a significant portion of the airfoil has stalled the analytical solutions will begin to lose accuracy, due to the polar files used. Therefore a stall analysis was performed that was similar to that done in the steady analysis.

After the time steps have run to a fully converged region the polar points for both blades of the airfoil are printed to the screen. The two blades are listed as m(jx,1) and m(jx,2), where the indexes of one and two refer to the blade number. This is an alteration from the steady analysis where the blade only required a single run at one location. Due to the unsteady nature of the flow entering the turbine, the flow must be examined for a full rotation of the blade over 360 degrees. It was found through this investigation that in unsteady flow the stall characteristics are not only a function of the wind velocity and location on the blade, but also on the azimuth angle as well.

In order to examine the dependence the three velocities at several yaw angles were examined. The velocities examined were 8 and 9 meters per second at 10 and 30 degrees of yaw, and 10 meters per second at 5, 10, 20 and 30 degrees of yaw. These velocities were chosen because 8 and 9 are located on the edge of stall, while the 10 meter per second range covers a wide range of yaw angles that all have stall in some form or another.

For the 8 meter per second range it was found that even at 10 degrees of yaw no stall had occurred on the blade of the turbine. At 30 degrees of yaw only a very small range of the airfoil was stalled over a very small range of angles as seen on figure III.6. The stall occurred between 32 and 35 percent of the chord length, which is very close to the range that the steady code first reached stall at 10 meters per second. The blade only seemed to stall near the zero degree blade axis when the yaw angle would have the greatest effect on the blade. What is surprising in this output is that it took a fairly large yaw angle of 30 degrees to cause the flow to separate, despite the fact that the flow is very close to the required stall velocity in steady flow.
The 9 meter per second range of velocities was somewhat more interesting than the previous data. As you recall under steady flow conditions the flow was separated, but not stalled from approximately .35 to .50 radius of the blade. The increase in yaw angle to ten degrees was found to have stalled the blade over a slightly larger region than the steady separated flow condition, as seen in figure B1. Note at this point that the stalled region is not symmetric, as the blade is stalled from 300 to 360 degrees and from zero to 45 degrees, a difference of 15 degrees. When the yaw angle is increased further to 30 degrees as in figure III.7 the maximum amount of blade stall is identical to the steady analysis at 10 meters per second, yet the asymmetry around zero degrees still exists.

So far the trends have been fairly similar to each other with no unexpected results. The maximum stall occurs when the blade is oriented straight up. As the azimuth angle increases the
stall dissipates, until it nears the vertical orientation again. Also the amount of stall has increased for every location examined so far.

The 10 m/s velocity range was examined to find out what happens when yaw is introduced to a flow velocity where stall already exists. At five degrees, the flow was stalled for all azimuth angles from .4 to .45 of the radius. Stalled flow was expected for this velocity, but the stalled region over an entire blade rotation was smaller than expected. Under steady conditions a much larger region was stalled, from approximately .3 to .55 of the blade. The maximum amount of stall matches exactly with the stall for the steady conditions, but the stall decreases from the steady amount for azimuth angle range of 45 to 300 degrees. The stall range is anti-symmetric as well. This is fairly interesting as the stall ranges for the velocities were not symmetric about the zero blade, or 180 degree angle axis. This means that the anti-symmetric flow is not a function of stall at all, but rather a function of the yaw input to the system.

When the yaw angle was increased to 10 degrees the blade once again developed a region where none of the blade was stalled, figure III.9. The maximum amount of stall remained unchanged, but the region where the maximum amount of stall occurred also increased. The anti-symmetry also reappeared with the unstalled region. Figures B2 and B3 show the stall plots for 20 and 30 degrees of yaw at 10 meters per second. At 20 degrees of yaw the maximum flow region increases, and the unstalled region increases in size by 30 degrees of azimuth angle. At 30 degrees the stalled region reaches its largest maximum yet, but the azimuth angles affected are decreased again by 30 degrees.
By examining the data collected for all yaw angles at ten meters per second some interesting trends are found. As the yaw angle increases less of the blade is stalled over the revolution cycle, but the section that is stalled becomes larger. The symmetry found for five degrees of yaw appears to be left over from the steady condition, because for all angles greater than 5 the flow is not symmetric about the zero-hundred eighty degree axis.

In order to understand how these alterations in stall are occurring, the orientations of the flow field must be examined. First, the S sequence of data is an upwind configuration meaning that the blades are upwind of the tower and hub of the system, meaning that the flow entering the blades is clean and uninterrupted. The yaw angle is defined as positive if from looking above the turbine blades are rotated clockwise from the flow. The azimuth angle definition states that blade 3 is at zero degrees when the blade is straight up, and the positive increase in angle is created with a counterclockwise rotation. This can be seen in figures III.10 and III.11.

Utilizing these two views some information can be gained about how the flow is interacting with the blade of the turbine. When the yaw angle is small the flow interaction is almost equal for all azimuth angles as the component of the flow along the blade at any blade position is very small. When the yaw angle is increased the maximum angle of attack that the blade interacts with is near the zero degrees orientation of the blade, as seen in figure III.11. As the blade rotates in the counterclockwise direction the horizontal effect of the flow along the
blade decreases, decreasing the effective angle of attack. This then causes the separated region of
the flow to decrease along the blade. This accounts for the effects seen in the stall figures in the
ten meter per second range. At five degrees of yaw the relatively small component of flow
moving parallel to the blade does not alter the angle of attack much during the full rotation. This
is why the stalled region decreases in size, but the entire rotation is stalled for some locations.
When the yaw angle is increased to ten degrees the parallel component of the flow is now large
enough that the effective angle of attack from 90 to 240 degrees of rotation has been decreased
enough to prevent stall. At the same time the angles close to the zero degree axis, have an
increased effective angle of attack, causing a slightly larger section of the blade to become
stalled.

7. Global Results

The first step in analyzing the exhaustive data sets that were created using the helicoidal
vortex model, is to examine the global results that are independent of the blade location. For this
analysis the two most important global contributions are the power and torque values. These
contributions can be analyzed by examining the data in two different ways. The first is to find the
average values for the power and torque and examine how alterations in the wind velocity and
yaw angle alter the terms. Secondly, the power and torque can be analyzed as a function of the
azimuth angle.

![Average Torque versus Velocity for Zero Yaw](image)

Figure III.12: Average Torque versus Velocity for 0 degrees of Yaw

Torque is often used in the examination of wind turbines, as it is fairly simple to measure
and is directly used in the generation of power. After the average values of torque were
computed using the sorting code the data was divided up to examine how changes in velocity
effected the torque output at a given yaw angle. The first of these curves can be seen in figures
III.12 and III.13. The solid line with solid square identifiers classifies the NREL data, while the
thin line with circular identifiers classifies the analytical average solution. What is quickly seen
is that the error for the torque is very small until a velocity of 10 meters per second. In both
curves the torque is slightly over predicted, but the error is actually less for the zero yaw
condition. It also is quickly noticed that the torque spike at 9 meters per second in the steady curve does not exist in the unsteady curve. This could be an effect of the yaw increasing the effective angle of attack over the blade, causing some sections to stall decreasing its efficiency. In figures B4 through B6, the average torque for 10 and 30 degrees can be found. What was found was that the error from 10 to 30 degrees increased greatly, yet the analytical solution was always greater than the actual torque value. Also for all the curves the error sharply increased after 10 meters per second, where a significant portion of the blade becomes stalled.

![Average Torque versus Velocity for 5 degrees of Yaw](image)

Figure III.13: Average Torque versus Velocity for 5 degrees of Yaw

The power is an obvious output of interest when discussing wind turbines, as that is the ultimate purpose of the turbine as a whole. In the unsteady analysis it was seen that the local power maximum occurred at 9 meters per second, and that the ultimate maximum was at 25 meters per second. Since the ultimate maximum occurs in a regime where the flow is fully stalled, and the pressure drag is providing all the torque so the maximum velocity analyzed was 15 meters per second. The average power for the NREL data can be calculated in one of two ways, by finding the average for all the rotations from the NREL cycle average data or by running the sorting code. After the average NREL data has been found the analytical power average is required. Like in the steady analysis this can be calculated using the equation:

\[ \text{Power} = \text{Torque} \times \Omega \]  

Therefore from this equation it is clear that the power curves for alterations of velocity at a given yaw angle will be identical to the torque curves, just at a different order of magnitude. For that reason the average power curves can be found in appendix B, figures B7-B10. Once again the curves are very accurate for yaw angles less than 30 degrees and velocities less than 10 meters per second.

This average analysis shows that the underlying factor that causes error in the torque and power curves is large amounts of separation over significant portions of the blade. Until a velocity where the flow starts to separate significantly the analytical solution was fairly accurate. While this is informative about the effects of velocity, the effects of yaw angle are hard to find in these figures. Due to this the power was also plotted versus the yaw angle for the velocities of 5,
7 and 10 meters per second, as seen in figure III.14. What is plotted in this figure is the power as a function of yaw angle. The first information gleaned from this curve is that approximately 2 kW is gained from an increase of 2 meters per second. Also the code consistently over predicts the power output for all velocities and yaw angles. When the curves themselves are examined it is clear that the best fit overall is for 5 m/s, where the error remains almost constant over the range of 30 degrees. The low flow velocity, and the lack of stall explain this. For 7 and 10 m/s, the error is not as consistent, although at 10 meters per second the maximum experimental power output occurs at a yaw angle of 10 degrees. This is a new outcome.

![Power versus Yaw Angle](image)

Figure III.14: Power versus Yaw Angle

Due to the fact that the airfoil stall is a function of the yaw angle the drop in stall from 5 degrees to 10 degrees actually increases the power output that occurs. By referring back to figures III.8 and III.9, this can be justified. At 5 degrees of yaw the entire blade is stalled over a small region. At 10 degrees of yaw a large portion of the blade is no longer stalled, and the stalled region has not increased significantly in size either. This means that more of the blade is near the optimal angle of attack increasing the torque and power output of the turbine. The error in the analytical solution would appear to be due to the post stall predictions in the polar file. Due to the fact that the polar decreases at a linear rate the effective decrease in angle of attack counteracts the stall, and increases the power output after the true maximum at 10 degrees.

8. Azimuth Angle Analysis

In the following sections the quantities of torque, dynamic pressure, and the coefficients of normal and tangential force will be analyzed as functions of the azimuth angle. Except for the torque, which is a global product, these terms are dependent on their span location, and therefore were analyzed at 30, 47, 63, 80, and 95 percent of the span.

All the quantities required some sorting and averaging in order to break them down into 360 degree average curves. Averaging the analytical solutions was a trivial affair as all the data was provided at identical azimuth angles over the entire run. By simply averaging each value at the proper azimuth angle, the 360 degree smooth solution was found. The experimental data was
slightly more difficult. Due to the fact that the data was taken at approximately every degree, but not exactly a simple average could not be made. In order to alleviate this problem the sorting program was used. Inside the sorting program a range was setup. All points within one half degree of an integer were averaged in order to create smooth solution over a full revolution of the turbine. This eliminated any discontinuities that were caused by the spread of the experimental data, and prevented the data from appearing too scattered. A few different ranges were applied, from .1 degrees to 1 full degree, but one half provided the best smooth average of the experimental data.

9. Torque

The torque in the analytical solution was found to not vary significantly when compared to the experimental alterations in the torque for low speeds and yaw angles. This can be seen in figure III.15.

![Figure III.15: Torque versus Azimuth Angle at 5 meters per second, 5 deg Yaw](image)

This is because according to the analytical solution the difference between the blades is minimal as neither blade is near a separation condition. Due to this fact no change in the torque would be seen, as the azimuth angle would have no effect on the torque. When the oscillation of the torque becomes noticeable is around 7 m/s, at 30 degrees of yaw, figure III.16.

![Figure III.16: Torque versus Azimuth Angle for 7m/s, 30 deg Yaw](image)
It is at this point that the analytical solution begins to oscillate. The oscillations between the analytical and experimental solutions do not seem to have much in common under these conditions. In the analytical curve the torque fluctuates about 5 percent from its average value. The peaks of the fluctuations occur at approximately 130 and 300 degrees of azimuth angle. While it does look there are two periods that developed here, it is actually only one as the distance between the maxima and minima are not constant. The experimental data on the other hand has a single visible mode. The experimental torque has only a single oscillation under most unsteady conditions and fluctuates at around one third its average value. This distinctly large difference in fluctuation would appear to be caused by the solution manner with which the analytical torque is found as it occurs under all conditions, whether stalled flow exists on the blade or not. When stalled flow does occur on the blade the torque no longer has a smooth nature, but rather reaches plateaus in the solution where the torque does not change much over a period of 10 to 20 degrees. A section where the torque on the blade is counteracting the other forces that are altering the torque coefficient would seemingly cause this. All the torque versus azimuth angles curves can be found in appendix C, where the experimental and analytical solutions are represented by square and circular identifiers respectively.

10. Dynamic Pressure

The dynamic pressure is a normalizing coefficient that is commonly used in aerodynamic design, due to the fact that it can be used to normalize experimental pressure values into coefficients of pressure. From the steady flow analysis it was clear that the dynamic pressure was well predicted in steady flow, and under most conditions this is true for the unsteady case as well. What was not clear from the unsteady analysis is that the dynamic pressure is a property of not only velocity and yaw angle, but of the span location as well. Variations between the analytical and experimental dynamic pressure indicate that there are differences between the theoretical and actual flow.

As stated previously for the NREL data the experimental dynamic pressure was found by finding the maximum pressure on the airfoil, and by making that pressure the normalizing pressure. Experimentally, the dynamic pressure is found from equation 15.

\[ Q_{\text{norm}} = 0.5 \rho V^2 \]  

(15)

Since the dynamic pressure is a function of the velocity it must be analyzed over a variety of spanwise locations, due to the velocity differences that occur due to rotation. In the following sections the trends found at all radial locations will be examined. The dynamic pressure figures for all cases examined can be found in appendix D. In appendix D solid and dashed lines represent the NREL data and analytical solutions, respectively.

Over all the velocities and yaw angles examined it was found that the dynamic pressure ranged from zero to approximately 375 Pa, as seen in appendix D. For low velocity ranges the dynamic pressure remained relatively constant altering only slightly. In these ranges the analytical data did an excellent job mimicking the experiment, and this continued into the higher ranges of attack and velocities. Only under large yaw angles of approximately 30 degrees did any major differences occur between the analytical and experimental solutions. At 6, and 8 m/s, and 30 degrees yaw a strange effect was noticed at 120 and 250 degrees. At these locations the experimental dynamic pressure value seemed to flatten for a short period and then carry on with the normal curvature, as seen in figure III.17. This can possibly be attributed to a separation bubble that appears on the blade at the large yaw angles, and relatively low speeds. A small
portion of the blade stalling could cause the dynamic pressure to remain momentarily constant and then, when the flow reattaches the curvature is renewed.

![30% Span Dynamic Pressure versus Azimuth Angle](image)

**Figure III.17: 6 m/s, 30 deg Yaw**

The error remains very low for all flows that are fully attached or only partially separated. Therefore the system does very well until 15 m/s, where a majority of the blade is separated causing the stagnation pressure to decrease near the peak of the dynamic pressure.

At the locations of 47, 63, 80, and 95 percent of the radius the resulting trends were very similar and can be discussed at once. For all of these locations the differences between the experimental and analytical solutions are very small for velocities less than 10 meters per second. It also was found that the dynamic pressure increases in magnitude with the velocity, as would be expected, but it also increases with increased yaw angle.

Obviously due to the radial location of the data points the dynamic pressure increases as the distance along the blade increases with time. Another global effect that is noticed is that for the high velocity conditions the experimental data is no longer smooth at the peak of the curve, but rather has a discontinuity or a flat region. This also occurs for some of the inner radial locations at lower velocities, but not the outer regions of 80 and 95 percent of the span. When the stall curves are compared with this data it is found that the flat region often corresponds with a region of stall on the blade. When the blade has become stalled the slope of the curve changes in the stalled region. This is why these slope discontinuities are noticed near 180 degrees, as this is where the flow is most often stalled. As the velocity increases the stalled region increases affecting more of the blade and dynamic pressure values.

Overall the relative accuracy of the dynamic pressure shows that there is excellent correlation between the analytical pressure values and the experimental pressure values well into the stalled region of flow. This then shows that very little deviation will be introduced into the analytical coefficient of pressure terms, which are defined by equation 16.

\[ C_p = \frac{P(r)}{Q_{norm}(r)} \]  

### 11. Coefficient of Normal Force, \( C_N \)

The coefficient of normal force allows the normal force on the blade under different conditions to be compared independent of what the magnitude of that force might be. Like in the steady analysis the experimental \( C_N \) is a function of the normalized chord line and the pressure
coefficients along the blade, see steady equation 6. The coefficient of normal force is a function of the lift and drag on the airfoil, as it makes a full rotation. During each cycle, the blade performs the normal force will vary, depending upon the inflow conditions. The normal force coefficient curves can be found in appendix D for all unsteady conditions.

For all the velocities and yaw angles analyzed there were some consistent results that were found. First, the normal force remains fairly constant as long as the yaw angle remains small, approximately 10 degrees or less. For yaw angles greater than 10 degrees a sinusoidal nature for the normal force becomes apparent, where the amplitude is a function of the yaw angle and the flow velocity. As the yaw angle and flow rate increases so does the amplitude of the oscillations in the normal force curves. The maximum normal force value occurs very close to 0 degrees of azimuth angle, for velocities and yaw conditions less than 10 m/s with a yaw angle less than 30 degrees. After this point is reached the flow becomes more random, and abstract, as opposed to the smooth sinusoidal curves previously seen. This would imply that large amounts of separation over the blade cause the flow to become uneven causing the normal force to become less dependent on the rotational components, and more dependent on three dimensional effects.

Another feature found in all the curves is that the minimum value occurs at 180 degrees of sinusoidal shape. This is the point when the blade is completely covering the hub and upper section of the tower. It is at this point that the blade’s velocity vector is exactly opposing the yaw component of velocity, decreasing the velocity over the blade, while at the same time decreasing its effective angle of attack. So not only are the lift and drag increased, but so is the angle of attack, which decreases the normal force, as the normal force is defined as in equation 17

\[ C_i \cos(\alpha) + C_d \sin(\alpha) \]  

As the effective angle of attack decreases, the lift and drag components of \( C_N \) become smaller. This also explains why the sinusoidal behavior becomes more prominent with increases in the yaw angle. With increases in either component, the differences caused by small changes in the angle of attack due to rotation become more prominent.

The normal force also changes as a function of span location as well. As the span length increases, the normal force and oscillation amplitude decrease for the analytic and experimental solutions. The two most notable differences that are found as the span length increases are changes in the Reynolds number and the velocity. Since the Reynolds number was assumed to be constant over the span length, this cannot be the cause. As the velocity increases the effect of the yaw flow input is decreased, so yaw velocities that alter the inner regions of the blade will have little effect on the outer regions, as the yaw flow component vector will be much smaller than the rotational velocity vector. The decrease in normal force as a function of increased span location is slightly more difficult to visualize. Recall that the coefficient of normal force is a function of the coefficients of lift and drag, which in turn are dependent on the pressure and dynamic pressure at a given location. As the span length increases, so does the lift as the disparity between the lower and upper surface pressures of the blade increase. At the same time the dynamic pressure is increasing as well as the velocity term in equation 15 is a function of the flow velocity and the rotational velocity \( \Omega \). Therefore since the normal force coefficient decreases with increased span length while the lift is increasing then it can be concluded that the dynamic pressure is increasing at faster rate. This increase in the normalizing factor would cause the \( C_N \) to decrease despite the fact that the normal force is increasing with increased span.

Overall the analytical \( C_N \) corresponded very well with the experimental data. Not only were the flow reactions similar, but they also had very similar solution values. While the flow
was fully attached the best results were found at 30 percent of the radius, as seen in figure III.18. In figure 28 the $C_N$ for 7$m/s$, and 10 degrees of yaw has excellent correlation with the experimental data. This is true for velocities of 5, 6, 7, and 8 m/s at all yaw angles with 30 percent radius. At 40 % of the radius the relationship between the analytical and experimental solutions is good, and holds true for the same regions of flow.

![Figure III.18: $C_N$ versus Azimuth Angle at 30% Radius for 7 m/s, 10 deg Yaw](image)

Holding along the same lines the $C_N$ at 63% of the radius was analyzed next. Here is where the differences due to radial location become noticeable. Unlike the two previous data sets analyzed, at 63 % the correlation between the analytic and experimental flow improves as the flow velocity increases, becoming very good at a velocity of 8 m/s, as seen in figure III.19. This excellent correlation remains true until a velocity of 11 m/s. This means that the best solutions were found as the flow approached separation, and just as minimal flow stall occurred over a small portion of the blade. What this seems to imply is that the analytical solution’s accuracy is best as the angle of attack approaches its $C_{L,\max}$, or optimal rotor efficiency. At inner locations like 30 and 47 percent of the radius this occurs at lower velocities, as stall is reached at 8 m/s, and 20 deg of yaw. As the blade radius is increased however higher velocities are required to induce stall, consequently causing the best correlation at higher velocities.

![Figure III.19: $C_N$ versus Azimuth Angle at 63% Radius for 8 m/s, 10 deg Yaw](image)

The final two radial locations examined in the validation process are 80 and 95 percent. At these two locations the high rotational velocity makes the stall conditions occur only at very
high velocities. Also at 95 percent there are three-dimensional effects that are also affecting the normal force. As previously noted the analytical solution has its best correlation as the angle of attack approaches the stall angle, as this causes the blade to operate at optimal conditions. The high rotational velocity also causes the stall speeds near the tip to require the entire blade to become stalled, which would cause some interesting three-dimensional effects, not to mention the fact that the theory is no longer valid once a significant portion of the blade is stalled. This would explain why at very high velocities like 11 meters per second the solutions have decent correlation, but the experimental data is very jumpy and almost random in nature. At these velocities, the sinusoidal nature is almost completely negligible, and the $C_N$ seems to change with no respect to the azimuth angle of the blade. This affect is shown in figure III.20.

![Figure III.20: $C_N$ versus Azimuth Angle at 80% Radius for 11m/s, 10 deg Yaw](image)

Overall the analytical solution did an excellent job in predicting the experimental solutions as seen in the previous figures and those in appendix D. What was found was that the best correlation occurred when the angle of attack of the blade was near the $C_{l,\text{max}}$ condition, and that the theory only broke down after stall occurred, which is most predominately seen for the velocities of 15 meters per second.

12. Coefficient of Tangential Force, $C_T$

The final coefficient to be analyzed as a function of the azimuth angle is the coefficient of tangential force. The tangential force is the force is the direction of the chord line as defined in figure II.5 of the steady analysis. $C_T$, like its counterpart $C_N$ can be defined by the coefficients of lift and drag.

$$C_T = C_l \sin(\alpha) - C_d \cos(\alpha)$$

(18)

Overall the correlations between the analytical and experimental solutions are very good for the tangential force coefficient. The first feature noticed is that the solution trends are almost identical. For small yaw angles, up to approximately 10 degrees, the tangential force remains relatively constant. For yaw angles greater than ten degrees $C_T$ becomes sinusoidal in nature like $C_N$. Also like the coefficient of normal force $C_T$'s minimum value occurs at approximately 180 degrees of azimuth angle. Once again the flow velocity causes this. When the blade is at 180 degrees, its velocity vector is exactly opposing the yaw velocity component of the flow. This causes the relative flow velocity to be a minimum at this point, along with decreasing the effective angle of attack. Since the flow velocity over the blade is at a minimum at 180 degrees the coefficients of lift and drag are at a minimum value for the rotation cycle. Therefore when the tangential force coefficient is calculated from equation 5, a minimum in the curve is a reasonable
result. An example of this can be found in figure III.21, which depicts the tangential force at 5 m/s with 30 degrees of yaw at a radial location of 47 percent.

![Figure III.21: CT versus Azimuth Angle at 47 % Radius, for 5 m/s, 30 deg Yaw](image)

Like the normal force the coefficient of tangential force is a function of the flow velocity, yaw angle and span location. In the following sections the effects of these parameters will examined in order to determine how they effect the analytical and experimental tangential force coefficients as seen in appendix D. In appendix D the dashed lines represent the analytical solution, while the slid lines represent the experimental data.

Flow velocity is the first parameter that affects the tangential force, and creates the largest changes with small alterations in magnitude. As the velocity increases for a given yaw angle and radial location, the tangential force increases as well. This continues to occur until flow separation occurs, which causes the theory to breakdown. Another interesting feature that appears is that the relative variation between the two solutions remains almost constant with changes in the flow velocity.

As an unsteady analysis the alterations to the tangential force coefficient due to the changes in yaw angles are especially important. The first effect that becomes clear is that with an increase in yaw angle, the amplitude of the sinusoidal action increases. Interestingly, the large difference that occurs is that the minimum value at 180 degrees decreases in magnitude with increased angle of attack, and the maximum value increases slightly. This would seem to make sense as the velocity over the blade decreases as more yaw is placed onto the blade decreasing the lift and drag. This pattern remains constant until the flow becomes stalled.

As a function of blade radius the tangential force reacts very similarly to the normal force. As the radius increases the tangential force coefficient decreases. This is due to the fact that the dynamic pressure increases faster than the coefficients of lift and drag that are used to find the tangential force, as explained during the $C_N$ section. Also as the blade radius increases the amplitude of the $C_T$ curves decreases. Since the amplitude is a function of the velocity change over the cycle, this means that the velocity difference between the blade and yaw flow becomes larger as the blade radius increases. This would seem to follow with the physics of the system as the rotational velocity $\Omega$ increases significantly with increased radius, making the yaw flow velocity small in comparison.
Overall the coefficient of tangential force correlates superbly with the experimental data, but this is somewhat a function of the radial location. Due to the velocity differences at various locations it is important to examine how and where these differences occur.

At thirty percent of the radius the tangential force matches the experimental data for all flow conditions less than 9 m/s. Interestingly as the yaw angle increases at these velocities the tangential force becomes more accurate. This effect is apparent in figures III.22 and III.23, which display the flow conditions of 7 m/s, at 10 and 30 degrees of yaw.

![Figure III.22: Cₜ versus Azimuth Angle at 30 % Radius, for 7 m/s, 10 deg Yaw](image1)

![Figure III.23: Cₜ versus Azimuth Angle at 30 % Radius, for 7 m/s, 30 deg Yaw](image2)

After nine meters per second the flow velocity becomes stalled for a portion of all conditions. This causes the sinusoidal nature to be lost, and the flow becomes much harder to predict and correlate between the analytical and experimental solutions. The region of accuracy also holds true for the flow at 47 and 63 percent of the radius. Yet when all of these flow locations are analyzed together it becomes apparent that the flow is still accurate for the unstalled azimuth angles, and only becomes inaccurate for the stalled regions.

The final two radial locations to be analyzed are the 80 and 95 percent regions of the blade. From the previous analysis it would be expected that the relative error between the curves would decrease as the flow velocity increases, and this is seen to a minimal extent. What differs most notably between these locations and those previously discussed is that they remain accurate
until over 11 m/s, figure III.24. This simply is a function of the stall nature of the blade. Since it
stalls at the inner regions at much lower velocities than the tips, this result was expected. One
unexpected feature of the high radial location curves is that high velocities the experimental
solution takes on a wavy nature that is not seen in the analytical solution. This is more than likely
a three-dimensional effect that is caused by the large amounts of stalled flow only a short
distance down the blade. It is possible that the stalled flow is increasing the three-dimensional
effects, and creating a less steady flow over the out regions of the blade.

Figure III.24: $C_T$ versus Azimuth Angle at 80 % Radius, for 11 m/s, 10 deg Yaw

Conclusions

Over the course of this analysis it has become clear that the helicoidal vortex model does
an excellent job in predicting the global and local parameters for a given wind turbine. By
comparing the experimental S sequence of data from the NREL’s Unsteady Aerodynamic
experiment it became clear that the power output and torque up to yaw angles of 30 degrees is
extremely accurate for such a computationally cheap analysis. The code, as was determined by
the background theory, was most accurate up to, or until just after stall occurred along the blade
of the airfoil. It was found that the stall initially occurs at 8 m/s, and 30 degrees yaw. Therefore it
was determined that all conditions with velocities and yaw angles less than this were fully
attached and therefore had good correlation with the experimental data. As the radial location
increased the dynamic pressure also increased decreasing the onset of stall until larger velocities
were reached. It is for this reason that at the locations of 80 and 95 percent of the radius the code
was accurate until 11 meters per second where stall starts to occur.

The coefficients of normal and tangential force were found to do an excellent job in
predicting the steady and unsteady flow in the pre-stall regions. Not only did the solutions have
the same orientations and curvature of the experimental data, but the solutions themselves were
very accurate, reaching the lowest error as the radial location approached $C_{1,max}$. For the inner
radial locations of 30, 47, and 63 percent one of the best cases occurred at 8 m/s, and 10 degrees
of yaw. For 80 and 90 percent the smallest error was found to occur at 10 m/s with around 10 to
20 degrees of yaw.
References

