# *MAE*

# *Ph.D. EXIT SEMINAR*

## Optimal Control of Tool/Workpiece Orientation for 5-axis CNC Milling with a Ball-end Cutter

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2:00pm~3:00pm
Bainer 2130
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When a ball--end milling tool cuts a given path on a smooth surface, it is desirable to maintain a fixed angle psi between the tool axis and the local surface normal at each point, to ensure a constant speed of the tool cutting edges against the surface. This means that the tool axis must lie on a cone of angle psi about the surface normal at each point, but its azimuthal position on this cone remains indeterminate. To resolve this indeterminacy, while minimizing actuation of the rotary axes that orient the workpiece relative to the tool, the component of in the surface tangent plane is specified through the parallel transport of a given initial state along the path. This amounts to the integration of coupled first--order differential equations that involve the Christoffel symbols for the given surface. Alternatively, the tool axis is shown to be rotation--minimizing with respect to the surface normal, and its orientation relative to the Darboux frame along the tool path can be determined by integrating the geodesic curvature along that path. The method is illustrated by closed—form solutions for simple analytic surfaces, and numerical integration using an object--oriented software implementation for free--form surfaces. The real--time implementation of such rotation--minimizing 5--axis tool motions for free--form surfaces is well within the scope of modern CNC systems.

The inverse kinematics problem (i.e., the determination of required inputs to the machine rotary axes) to achieve a rotation--minimizing tool motion is also addressed. In the context of an orientable--spindle machine, the results of the tool orientation control method are directly applicable to this inverse--kinematics problem. However, since they are expressed in terms of the integral of the geodesic curvature, a discrete time--step solution is proposed that yields accurate rotary--axis increments at high sampling frequencies. For an orientable--table machine, a closed--form solution that specifies the rotary--axis positions as functions of the surface normal variation along the toolpath is possible. In this context, however, the feasibility of a solution is dependent upon the surface normal along the toolpath satisfying certain orientational constraints. These inverse--kinematics solutions facilitate accurate and efficient 5--axis machining of free--form surfaces without "unnecessary" actuation of the machine rotary axes.